East Side Alliance
Mathematics Toolkit

2017-2018

East Side Alliance
East Side Union High School District
Alum Rock
Berryessa
Evergreen
Franklin-McKinley
Mt. Pleasant
Oak Grove
Orchard

http://www.svef.com/esa
The East Side Alliance Math Toolkit
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Why this Toolkit?

The East Side Alliance (ESA) consists of eight partner districts: the East Side Union High School District and the seven elementary partner districts: Alum Rock, Berryessa, Evergreen, Franklin-McKinley, Mt. Pleasant, Oak Grove, and Orchard. Together they serve approximately 85,000 students—almost the combined size of San Francisco and Oakland Unified School Districts. If the ESA were one district, it would be the third largest district in the state.

Over the past several years the East Side Alliance (ESA) has focused on implementing the mathematics Common Core State Standards (CCSS). The CCSS transforms what should be taught to students with a new emphasis on going deeper into mathematical concepts than perhaps might have been expected in the past. The CCSS has brought on a significant change in expected instructional practices to properly ensure that students in fact learn the content standards themselves. The CCSS implementation has generated changes in curriculum, instruction and assessment. Students are no longer only expected to know algorithms and formulas but also to think deeply and make sense of the mathematical practices they are learning. The new Smarter Balanced Assessments require students to explain their answers to reveal a deeper understanding of the mathematics involved. Helping students express their mathematical thinking has become an essential component of what students need to learn.

Learning mathematics has become increasingly important. Research shows that success in high school mathematics is a strong indicator of future success in college (Adelman). Success in math as early as third grade is a strong indicator of future success in school (Gingsburg). With this in mind, mastering the rigorous CCSS has become increasingly challenging. The purpose of this Toolkit is to provide a common focus on what instructional practices may be most helpful in helping students master grade level CCSS.

The ESA Math Toolkit was developed with a team comprised of both internal and external math experts, along with input from the most important expert of all: the classroom teacher. It is designed to be organic and potentially changing as we learn together what math instructional best practices look like. It is not intended to be exhaustive in nature but rather serve as a guide for districts and schools across the Alliance to help focus their instruction in the most effective way.
Acknowledgements

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East Side Alliance

Westside Silicon Valley Consortium

Santa Clara County Office of Education

Krause Center for Innovation

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We would also like to acknowledge the San Francisco Unified School District and their Math Toolkit model after which this ESA Math Toolkit is modeled.
Manny Barbara – CEO, SVEF

Manny Barbara serves as CEO for the Silicon Valley Education Foundation and is currently coordinating the East Side Alliance. He previously served for ten years as Superintendent of the Oak Grove School District. He has twice received the ACSA Region 8 Superintendent of the Year award; the Santa Clara County Alliance of Black Educators’ Hank Hutchins Award; the 100 Black Men Educator of the Year award and the state ACSA Robert E. Kelley award. He is married with four children, including two sons who are teachers and a third serving as a principal.

Margaret Bonanno – Consultant, SVEF

Margaret Bonanno spent thirty-two years working as a teacher, principal, director, and assistant superintendent in Santa Clara and Sonoma counties. Since 2000 she has been a lecturer in the Educational Leadership Departments at San José State University and CSU East Bay. Ms. Bonanno has coached over twenty-five beginning elementary, middle, and high school principals in strategies for successful leadership. For the last two years, she has been a member of a San José State University Teacher Education team implementing math education for teachers in the Franklin-McKinley School District. Her educational areas of focus are academic language and strategies for English Learner success. Margaret has become a continuing student of mathematics education. She has been a consultant with the SVEF since October 2016.

Ma Bernadette Salgarino – Mathematics Coordinator, SCCOE

Ma Bernadette Salgarino, Ed.D. is a Mathematics Coordinator of the Santa Clara COE. Bernadette has taught AP Calculus AB/BC, and AP Statistics among others, a mentor and an instructional coach at East Side Union High School District (ESUHSD). She is a National Board Certified Teacher, a member of the Mathematics Curriculum Framework Committee, and an AP Calculus Reader. Her classes had been videotaped for NBCT featuring instructional practices that support all learners to develop and strengthen literacy skills and mathematics understanding. She was awarded the Texas Instruments – STEM Teacher of the Year in 2012 and the ESUHSD Teacher of the Year in 2013.

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Marwa Yousofzoy is a Math Instructional Coach for Oak Grove School District in San Jose where she supports staff using research-based strategies. She holds a Master’s Degree in Education from UC Riverside and has spent the last ten years as a teacher, literacy facilitator, instructional coach, and teacher’s union officer. These positions led to a breadth of knowledge in how to effectively approach implementation, design meaningful curriculum and professional learning, and monitor student achievement through data analysis. She is an active member of local and county curriculum movements and supports SVEF and ESA in various capacities.
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Section 1
ESA Toolkit Anchor Documents

Section 1: Building a Dynamic Classroom Culture

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Guiding Principles for Mathematics Programs in California
Adapted from Mathematics Framework for California Public Schools

Five guiding principles underlie the Standards for Mathematical Practice, Standards for Mathematical Content, and other resources in this framework; see table IN-1. These philosophical statements should guide the construction and evaluation of mathematics programs in schools and the broader community. The Standards for Mathematical Practice are interwoven throughout the guiding principles.

Guiding Principle 1: Learning
All students will explore mathematical ideas in ways that will stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Guiding Principle 2: Teaching
All students will experience a mathematics program that is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence of learning.

Guiding Principle 3: Technology
All students will use technology as an essential tool that will be used strategically in mathematics learning.

Guiding Principle 4: Equity
All students will have a high-quality mathematics program that prepares them for college and careers.

Guiding Principle 5: Assessment
All students will experience many forms of assessment which will inform instruction and monitor student learning.

The above guiding principles are the foundation for the Standards for Mathematical Practice, Standards for Mathematical Content, and a variety of other resources in the California Mathematical Framework (www.cde.ca.gov). These principles have been slightly modified so that the focus of mathematics teaching remains clearly on the students. The National Council for the Teachers of Mathematics, Effective Mathematics Teaching Practices, strongly supports the California framework. In addition, the Teaching for Robust Understanding (TRU) Mathematics Framework supports the description for powerful classrooms.
Standards for Mathematical Practice
From Mathematics Framework for California Public Schools

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1) Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2) Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3) Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They
reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

4) Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5) Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6) Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with
quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7) Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$.

8) Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Overview

This document introduces the Teaching for Robust Understanding (TRU) framework. TRU provides a research-based response to the question, “What are the attributes of equitable and robust learning environments – environments in which all students are supported in becoming knowledgeable, flexible, and resourceful disciplinary thinkers?”

The answer, which resonates with what we know as teachers and researchers, appears in distilled form in Figure 1. The quality of a learning environment depends on the extent to which it provides opportunities for students along the following five dimensions:

1. The richness of disciplinary concepts and practices (“the content”) available for learning;
2. Student sense-making and “productive struggle”;
3. Meaningful and equitable access to concepts and practices for all students;
4. Means for constructing positive disciplinary identities through presenting, discussing and refining ideas; and
5. The responsiveness of the environment to student thinking.

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**Figure 1. The five dimensions of powerful classrooms.**

Complete TRU Math Conversation Guide can be found in the APPENDIX.
Section 2
Building a Dynamic Classroom Culture

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Setting up Positive Norms in Math Class

By Jo Boaler

Here are 7 of my favorite messages to give to students in math class, and some suggestions from youcubed as to how to encourage them:

- Everyone can learn math to the highest levels
- Mistakes are valuable
- Questions are really important
- Math is about creativity and making sense
- Math is about connections and communicating
- Math class is about learning not performing
- Depth is more important than speed
1. Everyone can learn math to the highest levels.

Encourage students to believe in themselves. There are different parts to this – first we need students to know that they can achieve at any math level, and there is no such thing as a math person. Brain information is really good for this.

Second we need them to have a “growth mindset” – believing that they can learn anything, and the more work they do the smarter they will get.

An important way to encourage a growth mindset is by praising what students have done and learned, not them as a person. So instead of saying “you are so smart”, say “it is great that you have learned that.”

Some videos you might want to share with students to encourage positive brain messages and a growth mindset:


[youcubed.org/students/boosting-messages](youcubed.org/students/boosting-messages)

What is a growth mindset?

There is a really damaging myth that pervades the US/UK and other countries – the idea that some people are born with a “math brain” and some are not. This has been resoundingly disproved by research but many students and parents believe this. It is really important to communicate “growth mindset” messages to students. Help them know that everyone is a math person and that the latest research is telling us that students can reach any levels in math because of the incredible plasticity of the brain.
2. Mistakes are valuable

Tell students that you love mistakes and that they will be valued at all times, tell them that it is good to make mistakes as we know that when people make mistakes, their brains are growing. This single message can be incredibly liberating for students. Here are some suggestions for encouraging positive thinking about mistakes:

1. Ask students with mistakes to present mistakes (especially deep, conceptual ones) on the board so that everyone can learn from them. If one student makes a conceptual mistake, there are probably many others making the same one.

2. When students get something wrong — instead of being discouraging or sympathetic, say “your brain just grew! Synapses are firing, that’s really good”

3. Ask students to read positive brain/mistake messages and choose their favorites that they will take on for the year. Eg “easy is a waste of time” “working hard grows your brain” “it is really important to make mistakes”. Ask them to draw brains with the messages on them that you can display on your walls, see right.

4. Crumpled Paper: Ask students to crumple a piece of paper and throw it at the board with the feeling they have when making a mistake. Then get them to retrieve the paper and color in all the lines, these represent synapses firing and brain growth from making a mistake. Ask them to keep the piece of paper in their math folders/notebooks to remind them of this.

Research shows that when students make mistakes, synapses fire and brains grow. Brain activity is particularly strong in individuals with a growth mindset. It is good to make mistakes.

3. Questions are really important.

Tell your students that you love questions about math and that they are really important. Research shows us that question asking is linked to high achievement – yet as students move through school they ask fewer and fewer questions, for fear of being thought clueless. You don’t need to be able to answer every question that students may come up with, sometimes it is good to say that you don’t know but you will find out, or ask other students if someone would like to answer the question.

Some suggestions for encouraging questions:

1. When good questions are asked, write them in large colored letters onto posters that you post around the room, to celebrate them. Show questions from a range of students.

2. Tell students they have 2 responsibilities in your classroom. One is to always ask a question if they have one, and the other it to always answer a question from classmates if asked.

3. Encourage students to ask questions – from you, other students and themselves, such as: why does that work? why does that make sense? Can I draw that? How does that method connect to another?

4. Encourage students to ask their own math questions. Instead of asking questions for them, give them interesting mathematical situations and see what questions arise for them.

In studies, student question asking has been shown to steadily decline as students go through the grades in the US, showing this relationship:
4. Math is about creativity and making sense.

The key to understanding math is making sense of it. Many students believe that math is a set of formulas that have to be remembered - this belief is associated with low achievement. Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

Some methods for encouraging sense making and creative math:

1. Always ask students – why does that make sense? Ask this whether their answers are correct or incorrect.

2. Encourage visual mathematics. Ask students to draw their solutions. Ask them to think about how they see math. In this video (http://youtu.be/1EqrX-gsSOg) Cathy Humphreys asks students to make sense of 1 divided by 2/3 by drawing their solutions.

3. Show mathematical ideas through visual representations. All mathematics can be represented visually, and visual representations give many more students access to understanding. We have many examples of visual mathematics on youcubed and in the classroom video above.

4. Use number talks that value students’ different ways of seeing math and solving problems. This video teaching number talks also shows visual solutions. http://youcubed.org/teachers/2014/from-stanford-onlines-how-to-learn-math-for-teachers-and-parents-number-talks/

5. When students finish questions, ask them to think of new, harder questions. These could be questions to give to other students. This is a really good strategy for differentiation.
5. Math is about connections and communicating.

Math is a connected subject, but students often think it is a set of disconnected methods. We made a video to show some connections and students loved it.

yousubed.org/students/a-tour-of-mathematical-connections

Mathematics is a form of communication, some people think of it as a language. Some strategies for encouraging connecting and communicating are:

1. Show the connections video.
2. Encourage students to represent their math results in different forms eg words, a picture, a graph, an equation, and to link between them, see below.
3. Encourage color coding, ask students to show with color where a mathematical idea is, see below.

\[ y = 4x + 1 \]

- There is one square at stage 1.
- For each additional stage a square is added adjacent to the original on all four sides.
- The figure continues to grow to the left, right, up, and down, adding four squares for each new stage.
6. Value depth over speed.

Many people incorrectly believe that being good at math means being fast at math. It doesn’t and we need to dissociate math from speed. When we value fast computation (as many classrooms do) we encourage a subset of learners who compute quickly and discourage many others, including deep slow thinkers who are very important to math (see sidebar).

We no longer need students to compute fast (we have computers for this) we need them to think deeply, connect methods, reason, and justify.

1. Tell students you don’t value fast work. Mathematical thinking is about depth not speed.
2. Don’t let mathematical discussions be driven by the fastest students.
3. When asking for hands up, don’t always take answers from the fastest students.
4. Don’t use flash cards, speed competitions, timed tests, instead value depth, creativity, different ways of thinking about math, and different explanations. A paper showing the research suggesting timed tests cause math anxiety is here: http://youcubed.org/pdfs/nctm-timed-tests.pdf

“I was always deeply uncertain about my own intellectual capacity; I thought I was unintelligent And it is true that I was, and still am, rather slow. I need time to seize things because I always need to understand them fully. Towards the end of the eleventh grade, I secretly thought of myself as stupid. I worried about this for a long time.

I’m still just as slow. (...)At the end of the eleventh grade, I took the measure of the situation, and came to the conclusion that rapidity doesn’t have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other. This is where intelligence lies. The fact of being quick or slow isn’t really relevant.”

- Laurent Schwartz,
Winner of the Fields Medal
(A Mathematician Grappling with His Century, 2001)
7. Math class is about learning, not performing.

Many students think that their role in math class is not to learn but to get questions right – to perform. It is important for them to know that math is about learning, and to know that math is a growth subject, it takes time to learn and it is all about effort. Some strategies for making math a learning, not a performing subject:

1. Grade and test less. Math is the most over-graded, over-tested subject in the curriculum. Neither grades nor tests have been shown to increase learning, from research, and both make students feel they are performing and not learning. Grades often make students think they are a reflection not of what they have learned but who they are. There is a video reflecting this at [http://youtu.be/eoVLBEuqB0](http://youtu.be/eoVLBEuqB0)

2. Instead, give diagnostic comments. These take longer but are extremely valuable and can be done less often.

3. Use “assessment for learning” strategies (see sidebar).

4. If you have to grade, then give grades for learning, not for performing eg for asking questions, representing ideas in different ways, explaining work to others, making connections. Assess the breadth of math, not just a small part of math – procedure execution.

5. You may have to give grades to your administration but that doesn’t mean you have to give them to the students. Grades communicate fixed messages about learning and are often counter-productive for student

Assessment for learning (A4L) teaching strategies have been shown to drastically increase student achievement, if they are used instead of summative tests and grades. It has been estimated that if teachers in England used A4L strategies the achievement of their students would increase so much the country would move, in international comparisons, from the middle of the pack to the top 5 (Black and Wiliam, 1998). At [http://yucubed.org/teachers/ wp-content/uploads/2014/08/ Formative-Assessment-Strategies. pdf](http://yucubed.org/teachers/wp-content/uploads/2014/08/Formative-Assessment-Strategies.pdf) we are sharing our favorite A4L strategies.
Positive Norms to Encourage in Math Class

By Jo Boaler

1. Everyone Can Learn Math to the Highest Levels.
Encourage students to believe in themselves. There is no such thing as a “math” person. Everyone can reach the highest levels they want to, with hard work.

2. Mistakes are valuable
Mistakes grow your brain! It is good to struggle and make mistakes.

3. Questions are Really Important
Always ask questions, always answer questions. Ask yourself: why does that make sense?

4. Math is about Creativity and Making Sense.
Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

5. Math is about Connections and Communicating
Math is a connected subject, and a form of communication. Represent math in different forms eg words, a picture, a graph, an equation, and link them. Color code!

6. Depth is much more important than speed.
Top mathematicians, such as Laurent Schwartz, think slowly and deeply.

7. Math Class is about Learning not Performing
Math is a growth subject, it takes time to learn and it is all about effort.
Establishing Class Norms

What is this structure?
Class norms are a set of statements of value or behavior that support active and equitable participation in the classroom. They set expectations for all members of the classroom community, adults and students. Class norms are stated in a positive tone, apply to all situations all the time, and are few in number.

Why do I use this structure?
Class norms help students, and give them opportunities, to be accountable to the community and to act on the prosocial values of responsibility, respect, fairness, caring, and helpfulness. They support equitable participation, engagement with the practices of doing math, and establishment of a positive classroom climate.

When do I use this structure?
Establish classroom norms at the beginning of the school year and continue to reinforce them every day.

How do I use this structure?

Establishing class norms:
❖ Hold a discussion of what the community wants the classroom to look and feel like. The norms could come out of this discussion, or this discussion could lay the groundwork for discussion of teacher-chosen norms.
❖ It takes time and effort for students to follow the norms consistently. Provide clear expectations and examples of what norms look like. Have students brainstorm and share your own powerful vision.
❖ Distinguish between rules and norms. Rules, which establish classroom safety and efficiency, can support norms. Rules should be established by the teacher on the first day of school. Phrasing the rules using a positive tone contributes to a sense of community.
❖ Give, or have students generate, specific examples of what the norms look like and sound like.

Reinforcing class norms:
❖ Continue to remind and reinforce the positive behaviors that you see throughout the year. Include specific examples of what the norms look like and sound like.
❖ Participation Quiz and Groupwork Feedback strategies can be used when establishing the norms and, as the year continues, to support attention to the norms.
❖ Encourage students to notice themselves and others supporting the norms. Celebrate positive mathematical interactions.

Sample Math Class Norm Poster

Errors are gifts that promote discussion.
Answers are important, but they are not the math.
Talk about each other’s thinking.
Use multiple strategies and multiple representations.

Ask questions until ideas make sense.

SFUSD Mathematics Department. June 2015. sfusdmath.org
Establishing Class Norms (continued)

Sample Class Norms

Choose 4-6 norms to use with your class, including one or two from each of the following value categories, depending on the needs of your students. Note that some norms support multiple values. Make the norms your own by discussing them with your students and changing the language if necessary.

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<th>Norms</th>
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<tr>
<td>Collaboration</td>
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<tr>
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<td>❖ We will help each other.</td>
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<tr>
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<td>❖ Ask questions and share ideas.</td>
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<td>❖ Members of your team are your first resource.</td>
</tr>
<tr>
<td></td>
<td>❖ Smarter together than apart.</td>
</tr>
<tr>
<td></td>
<td>❖ You have the responsibility to ask for help, and you have the</td>
</tr>
<tr>
<td></td>
<td>responsibility to offer it.</td>
</tr>
<tr>
<td></td>
<td>❖ No one is done until everyone is done.</td>
</tr>
<tr>
<td>Growth Mindset</td>
<td>❖ Errors are gifts that promote discussion.</td>
</tr>
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<td></td>
<td>❖ Answers are important, but they are not the math.</td>
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<tr>
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<td>❖ We learn from mistakes.</td>
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<tr>
<td></td>
<td>❖ We learn from trying new things.</td>
</tr>
<tr>
<td></td>
<td>❖ We learn from taking risks.</td>
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<tr>
<td>Communication</td>
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<tr>
<td></td>
<td>❖ Use multiple strategies and representations.</td>
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</table>

SFUSD Mathematics Department, June 2015, sfusdmath.org
Collaborative Learning Guide

Collaborative learning gives the responsibility of the learning to the students by using groups and pairs of students to fulfill a task or assignment within the classroom. The Common Core Math Practice Standard 3 calls for students at all grades to listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

<table>
<thead>
<tr>
<th>Creative Effective Collaborative Activities</th>
<th>Within a Collaborative Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ Is the activity highly structured physically, spatially and temporally?</td>
<td>➢ Students are invested in their own learning.</td>
</tr>
<tr>
<td>➢ Do students know the rationale for the activity?</td>
<td>➢ Learners actively participate.</td>
</tr>
<tr>
<td>➢ What are the teacher’s academic and behavioral expectations of the students?</td>
<td>➢ Teachers become learners at times, and learners sometimes teach.</td>
</tr>
<tr>
<td>➢ How will the learning activity affect motivation?</td>
<td>➢ Respect is given to every member.</td>
</tr>
<tr>
<td>➢ Does the activity accommodate various group speeds?</td>
<td>➢ The project / question should be of interest and challenging to the students.</td>
</tr>
<tr>
<td>➢ How long will the activity take?</td>
<td>➢ Diversity is celebrated and all contributions are valued.</td>
</tr>
<tr>
<td>➢ Is there an opportunity for students to process how the group functioned and the learning that occurred during the activity?</td>
<td>➢ Students learn skills for resolving conflicts when they arise.</td>
</tr>
<tr>
<td>➢ Does the structure of the activity give students time to process the new information before they are asked to respond?</td>
<td>➢ Members draw upon their past experience and knowledge.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size</th>
<th>Formation</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest group is 2. Largest recommended group is 6. Smaller groups will require fewer social skills and will work more quickly. Larger groups generally generate more ideas, deal better with complex ideas and create fewer group reports for the teacher to process.</td>
<td>Heterogeneous grouping with regards to academic achievement, task orientation, ability and learning style can be used depending on the subject matter or collaboration technique used. If the project is long or detailed then the support of a stronger academic student in each group will help complete the project. Student self-selection is generally not successful, although students can provide input for the teacher to consider. Random assignment promotes the idea that everyone is expected to work with everyone else at some point. Random is best used if the task is of short duration.</td>
<td>Groups created for longer projects should be structured carefully. Groups that stay together for long periods (4-6 weeks) are more likely to form stronger bonds, develop more complex collaborative skills and can tackle more complex tasks. Groups should stay together long enough to feel successful, but not so that they become counter-productive. Avoid breaking groups up because they are having trouble functioning since the group will feel unsuccessful and transfer that to the next group. They need to persevere.</td>
</tr>
</tbody>
</table>
## Possible Student Roles within the Group

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Facilitator</strong></td>
<td>Keeps group on task and verifies that all contribute.</td>
</tr>
<tr>
<td><strong>Recorder</strong></td>
<td>Takes notes on important thoughts expressed in the group. Writes final summary.</td>
</tr>
<tr>
<td><strong>Reporter</strong></td>
<td>Shares summary of group with large group. Speaks for the group, not just personal view.</td>
</tr>
<tr>
<td><strong>Materials Manager</strong></td>
<td>Picks up, distributes, collects, turns in, or puts away materials.</td>
</tr>
<tr>
<td><strong>Time Keeper</strong></td>
<td>Keeps track of time and reminds group how much time is left.</td>
</tr>
<tr>
<td><strong>Checker</strong></td>
<td>Checks for accuracy and clarity of thinking during discussions. Checks written work and tracks points.</td>
</tr>
</tbody>
</table>

## Group management Tips

<table>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Noise</strong></td>
<td>Develop and practice a “QUIET or Zero-noise” signal. Brainstorm what that would be with the students. Practice appropriate internal and external voices.</td>
</tr>
<tr>
<td><strong>Deadlines and Task Structure</strong></td>
<td>Give students specific tasks to finish within a predetermined time limit. Use a timer.</td>
</tr>
<tr>
<td><strong>Instructions</strong></td>
<td>Show, don’t tell, instructions (have a group model the steps). Have students tell each other the instructions to make sure they understand prior to starting the task.</td>
</tr>
<tr>
<td><strong>Questions</strong></td>
<td>Answer team questions only. Individual questions should be handled within the team. Use the “3 Then Me” technique.</td>
</tr>
<tr>
<td><strong>Circulate</strong></td>
<td>Use proximity. Monitor discussions to check for understanding and to be aware of collaborative skills that may need to be addressed.</td>
</tr>
</tbody>
</table>

Excerpts from *Speaking and Listening Learning Stations, Grades 6 – 8* by Schyrlet Cameron and Suzanne Myers
Collegial Discussion

Common Core State Standards for Speaking and Listening item 1 (CCS–SL.1) calls for students to initiate and participate effectively in a range of collaborating discussions with diverse partners. They are to work with peers to promote civil, democratic discussion and decision-making, set clear goals and deadlines, and establish individual roles as needed. CCS–SL.1 also calls for students to follow rules for collegial discussions. Collegial discussions are mutually respectful conversations between student colleagues in a group or classroom environment.

Excerpts from Speaking and Listening Learning Stations, Grades 6 – 8 by Schyrlet Cameron and Suzanne Myers

<table>
<thead>
<tr>
<th>Discussion Guidelines and Skills</th>
<th>Discussion Sentence Stems</th>
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<tr>
<td><strong>When speaking, participants strive to .....</strong></td>
<td>Sentence starters for students to facilitate a safe and cooperative classroom or group discussion.</td>
</tr>
<tr>
<td>▪ sustain a main idea</td>
<td><strong>Agreement</strong></td>
</tr>
<tr>
<td>▪ be original with interesting, thought-provoking ideas.</td>
<td>▪ “I agree with ________ because ________.”</td>
</tr>
<tr>
<td>▪ have quality in their comments.</td>
<td>▪ “I like what ________ said because ________.”</td>
</tr>
<tr>
<td>▪ include textual references– the more specific the quotation, with reference to page and paragraph numbers, the better.</td>
<td>▪ “I agree with ________ because ________ then on the other hand ________.”</td>
</tr>
<tr>
<td>▪ make reference to other works.</td>
<td><strong>Disagreement</strong></td>
</tr>
<tr>
<td>▪ maintain the accuracy of their comments.</td>
<td>▪ “I disagree with ________ because ________.”</td>
</tr>
<tr>
<td>▪ question for greater understanding.</td>
<td>▪ “I’m not sure I agree with that because ________.”</td>
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<td><strong>When listening, participants strive to .....</strong></td>
<td>▪ “I can see that ________ however, I disagree with (or can’t see)”</td>
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<td>▪ listen to other students and not be “checked out”.</td>
<td><strong>Clarifications</strong></td>
</tr>
<tr>
<td>▪ see how the comments fit...follow the flow of the discussion.</td>
<td>▪ “Could you please repeat that for me?”</td>
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<td>▪ be able to reference previous comments.</td>
<td>▪ Paraphrase what you heard and ask, “Could you explain a bit more, please?”</td>
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<td>▪ listen for greater understanding.</td>
<td>▪ “I’m not sure I understood you when you said ________ Could you say more about that?”</td>
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<td>▪ wait patiently for the speaker to finish before sharing ideas.</td>
<td>▪ “Is there evidence for the position?”</td>
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<td><strong>In a collegial conversation, participants .....</strong></td>
<td>▪ “How does that support our work/mission at ________?”</td>
</tr>
<tr>
<td>▪ are consistent in participation.</td>
<td><strong>Confirmation</strong></td>
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<tr>
<td>▪ show leadership— students help others to enter the discussion.</td>
<td>▪ “I hear ________”</td>
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<td>▪ show empathy.</td>
<td>▪ “I believe ________”</td>
</tr>
<tr>
<td>▪ have the ability to learn and adjust to the dynamics of the class.</td>
<td>▪ “I discovered ________”</td>
</tr>
<tr>
<td>▪ incorporate politeness and respect for all members of the class.</td>
<td>▪ “I learned that ________.”</td>
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<td>▪ maintain eye contact and call others by their names.</td>
<td><strong>Confusion</strong></td>
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<td>▪ show patience with the process. (It takes some time to develop a group dynamic where everyone feels at ease.)</td>
<td>▪ “I don’t understand ________.”</td>
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<td>▪ demonstrate preparedness— books and articles are marked, responses are written, questions are prepared.</td>
<td>▪ “I am confused about ________.”</td>
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<td>▪ students are willing to state own ideas even if different from those of other students or the teacher.</td>
<td>▪ “Can you explain that another way?”</td>
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<td><strong>Extension</strong></td>
<td>▪ “I have a question about ________.”</td>
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<tr>
<td>▪ “I was thinking about what ________ said, and I was wondering what if ________.”</td>
<td><strong>Review</strong></td>
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<td>▪ “This makes me think ________.”</td>
<td>▪ “I want to go back to what ________ said.”</td>
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<td>▪ “I want to know more about ________.”</td>
<td>▪ “I like ________.”</td>
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<td>▪ “Now I am wondering ________.”</td>
<td>▪ “I noticed that ________.”</td>
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Stepping Stone Planning Protocol

Teaching grade-level math through problem-solving and supporting students to become independent learners (20-60 min.)

| Pre-Planning | ● Select a Good Problem  
|              |   ○ At or bridging to grade-level standards  
|              |   ○ Open-ended with entry points at different levels of understanding  
|              | ● Prepare to Discuss Solutions  
|              |   ○ Cultivate a safe environment where students feel comfortable asking questions and sharing their ideas  
|              |   ○ Anticipate and prepare for a variety of approaches and strategies that students might use (the “stepping stones”)  
|              |   ○ Strategize on ways to facilitate student ideas toward grade-level standards  
| Pose/Unpack the Math Problem | ● Present the problem to the class; clarify the language and context  
| (1-5 min.) | ● For a complex problem, invite student interaction (turn and talk, restate, justify high and low possible answers, predict, share what they notice etc.)  
| Write Silently | ● Give students silent time to independently arrive at their process and solution (write, sketch, solve, represent, diagram, etc.)  
| (1-4 min.) | ● If stuck, have students write what they do know and circle areas of difficulty  
| Discuss in Pairs | ● In pairs, students collaboratively share solutions and areas of difficulty  
| (2-6 min.) | ● Use proximity: look, listen, lead, and leave  
| | ● Identify 3 strategic solutions for “share” step, each progressing toward grade-level standards  
| Share Solutions | (This step is often combined with Polish Ideas.)  
| (4-15 min.) | ● Students (or teacher) present(s) and explain(s) solutions  
| |   ○ “Audience” practices collaboration skills: eye-contact, nodding, asking clarifying questions, building on ideas, and using names  
| | ● Praise the process (not the person)  
| | ● Keep visuals of board work for ongoing reference  
| Polish Ideas | ● Facilitate discussion using work and visuals:  
| (2-10 min.) |   ○ Uncover/+∆ (strengths and challenges) in approaches  
| |   ○ Affirm and celebrate learning from mistakes  
| Summarize Learning | ● Summarize salient points at grade-level standard.  
| (5-10 min.) | ● Model, and make explicit: precision, correct tools, conventions, abstract reasoning, structures, generalizations, language for affirming, critiquing, and arguing  
| | ● Reinforce the lesson objective, and supplement with “Mini Lecture” if necessary  
| Debrief the Process | ● Facilitate a brief student discussion on what worked for students and what was challenging about the content and lesson components  
| (5-10 min.) | ● Reflect on your own learning as a teacher  
| Post-Lesson Reflection | ● In what ways did you support students cognitively, socially and/or emotionally?  
| | ● What are some next steps?  

Stepping Stone Planning Protocol—Revised 2016-06-01 DRAFT  
Source: Kevin Drinkard; Inspired by teachers in Mt. Pleasant ESD and Phil Daro.  
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Key considerations of strategy: Stepping Stone Protocol

These considerations are intended to support teachers to more successfully integrate the Stepping Stone Protocol into their repertoire (Informed from work with teachers in Mount Pleasant Elementary School District).

Consideration 1: Advance Planning. Student approaches to problems are the “stepping stones” in this protocol, and the teacher uses them to take students from where they are to where they need to be (standard-level mathematics). It takes time and advanced planning to: find a good problem, anticipate variety of student approaches, and think in advance how to build from those approaches to facilitate standard-level understanding.

TRU reference:
1. The Mathematics: Provide opportunities for building a coherent view of mathematics
2. Cognitive Demand: scaffolding and productive struggle that is “just right”: struggle vs. frustration can be a fine line
3. Uses of Assessment: Student solutions (as they discuss them, write about them) provide formative assessment to guide instructional choices

Consideration 2: This style of teaching is developmental – for the teacher and for the students: If students and/or the teacher are not used to this style of instruction, there may be some frustration at first - an “implementation dip.” It may be strategic to begin with content that is less cognitively demanding, then build up over time.

TRU reference:
1. Cognitive Demand: Begin with more scaffolds; over time, provide opportunities to grapple with more sophisticated content and with fewer scaffolds
2. Access to Mathematical Content: Participation structures (individual, group, whole-class) engage an increasing number of students over time
3. Agency, Authority, and Identity: Increasingly, students demonstrate ownership of their ideas; it becomes a class norm to build on each others’ ideas

Consideration 3: Collaborate with colleagues/coaches: If this style of instruction is new, form collaborative partnerships (peer coach, teacher-buddy, PLCs, Lesson Study Groups, etc.) to identify focus for professional growth, solicit resources/support/colleagues, practice and reflect

Variations:
The Stepping Stone Protocol is flexible:
- Quick Use: During warm-ups, use it like a write-pair-share (10 minutes)
- In-depth use: Build a whole-class lesson using a more sophisticated problem (like a MARS task or SBAC Performance Task)
Facilitation Guide for Whole Group Instruction in Math Class

Consider the strategies below when planning to encourage and support math talk in a lesson. Select one or two strategies for a lesson:

● **Provide think time and wait time**
  - allow students time to think quietly for a minute before asking them to respond
  - after you’ve asked a question, wait at least 10 seconds before calling on anyone (“keep the answer in your head”)

● **Vary the modes of response: give students options for how to respond to your questions, such as:**
  - “show it on your fingers”
  - “turn to a partner and whisper the answer”
  - “keep it in your head”
  - “raise a quiet hand”
  - “tell your partner your answer, then ask: ‘do you agree or disagree?’”
  - “put your thumb to your chest when you have a strategy”
  - “all together…”
  - “once you have an answer, try to think of another way to solve it”
  - response cards (students hold up prepared cards with “true” “false”; numbers; “A”, “B”, “C”, “D”; or another appropriate answer

● **Ask students to think and talk about each other’s math**
  - “who solved it exactly the same way?”
  - “raise your hand if you understand exactly how ___ solved it”
  - “what do you think ___ was thinking when s/he solved it this way?”
  - “do you agree or disagree? Why?”

● **Encourage student---to---student conversations**
  - Ask each participant to call on the next speaker (“choose someone who has not had a turn to speak yet”)
  - Ask: “does anyone have a question for ____?” and allow them to call on each other to ask and answer questions
  - Remind students to make eye contact with the person they ask or call on

● **Offer sentence frames on a poster or sentence strip**
  - “I agree with ____ because…”
  - “I disagree with ____ because…”
  - “I think ____ solved it like that because…”
  - “___’s idea is interesting because…”

LCI Math
Five Productive Talk Moves

Each talk move listed below is a suggested action that is found to be effective for making progress toward supporting thinking and learning. Each move can be used to support productive talk and also establish a status-free classroom environment in which all students have an equal voice.

<table>
<thead>
<tr>
<th>Talk Move</th>
<th>Definition/Ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revoicing Teacher revoices</td>
<td>• This talk move allows teachers to interact with a student who is unclear.</td>
</tr>
<tr>
<td>“So you’re saying that it’s acting as a catalyst?”</td>
<td>• The teacher tries to repeat some or all of what the student has said.</td>
</tr>
<tr>
<td></td>
<td>• Then the teacher asks the student to respond and verify whether or not the revoicing is correct.</td>
</tr>
<tr>
<td></td>
<td>• Few students will improve if teachers only call on the students who are easy to understand.</td>
</tr>
<tr>
<td></td>
<td>• This move can also be effective when the teacher understands what a student has said but is not sure that the other students in the class understand.</td>
</tr>
<tr>
<td>Restating (Repeating)</td>
<td>• This talk move extends the responsibility to the students in the classroom.</td>
</tr>
<tr>
<td>Student restates.</td>
<td>• By asking one student to repeat or rephrase what another student has said, it requires the class to listen to each other and make sense of ideas shared.</td>
</tr>
<tr>
<td>“Can you repeat what he just said in your own words?”</td>
<td>• After the student has restated, go immediately back to the original student for follow up.</td>
</tr>
<tr>
<td></td>
<td>• This move also allows the rest of the class to hear the idea shared one more time so they have more time to process what was shared.</td>
</tr>
<tr>
<td>Agree/Disagree (Reasoning)</td>
<td>• This move encourages students to apply their own reasoning to someone else’s reasoning.</td>
</tr>
<tr>
<td>“Do you agree or disagree and why?”</td>
<td>• The teacher should not support one position or another but should use the talk move to elicit other ideas.</td>
</tr>
<tr>
<td></td>
<td>• It is critical that students support their decision by explaining their reasoning as it supports students’ learning.</td>
</tr>
<tr>
<td>Add on</td>
<td>• This move increases participation by asking other students to add comments to previous statements.</td>
</tr>
<tr>
<td>“Who can add an idea to this discussion?”</td>
<td>• This prompting for more input on previous statements will, over time, result in students showing more willingness to weigh in on what the group is considering.</td>
</tr>
<tr>
<td>Wait Time</td>
<td>• This move is not about talking at all but instead about silence.</td>
</tr>
<tr>
<td>Wait Time 1</td>
<td>• Wait Time 1: Occurs after a teacher has posed a question and before accepting a response.</td>
</tr>
<tr>
<td>Wait Time 2</td>
<td>• Wait Time 2: Occurs after a student response has occurred and before the teacher or other students respond or comment.</td>
</tr>
<tr>
<td></td>
<td>• The amount of time to wait should be determined based on what the students must think about.</td>
</tr>
</tbody>
</table>


*Thinking Through Quality Questioning: Deepening Student Engagement* by Jackie A. Walsh and Beth D. Sattes
The SAMR Model is a tool designed by Dr. Ruben Puentedura to help educators integrate technology in their classes and to understand the levels of rigor involved in that work. The model's design allows educators to build their capacity to better understand the complexity involved in successfully implementing 1-to-1 technology or other forms of technology in the classroom. The components of SAMR Model include:

- Students create meaning out of what they learn.
- Students organize information so they can create mental models.
- Students integrate individual skills into whole sets of processes.
- Students apply what they have learned to new or novel situations.
### SAMR Model Planning Template

<table>
<thead>
<tr>
<th>SAMR Levels Bloom’s and/or Webb’s DOK</th>
<th>Levels Defined</th>
<th>Examples from the classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Redefinition</strong>&lt;br&gt;Extended thinking&lt;br&gt;Creating &amp; Evaluating</td>
<td>Technology allows for the creation of new tasks that were previously inconceivable. Student cognitive processes require increased (including highest levels of) critical thinking (higher levels of Bloom’s Taxonomy or Webb’s Depth of Knowledge or DOK). This level may include 4 of the 5 1-to-1 Frameworks.</td>
<td>Student work requires technology use so that students can complete tasks previously not available (tasks that would otherwise be inconceivable for teachers or students).</td>
</tr>
<tr>
<td><strong>Modification</strong>&lt;br&gt;Strategic thinking&lt;br&gt;Analyzing</td>
<td>Technology allows for significant task redesign. Student cognitive processes require increased critical thinking (various levels of Bloom’s Taxonomy or Webb’s Depth of Knowledge)</td>
<td>Student work requires technology to be used and because of this, there is a significant redesign of student tasks (the technology supports high levels of critical thinking-Blooms or Webb’s DOK-and enables 21st century learning opportunities).</td>
</tr>
<tr>
<td><strong>Augmentation</strong>&lt;br&gt;Skills &amp; concept development&lt;br&gt;Applying</td>
<td>Technology acts as direct tool substitute with functional improvement. Students must use the technology, but the work includes limited critical thinking skills (various levels of Bloom’s Taxonomy or Webb’s Depth of Knowledge).</td>
<td>Student work requires technology to be completed and the requirements of the tasks students are being directed to do generally require increased critical thinking with technology acting as a learning tool (the technology supports the student tasks or enables 21st century learning opportunities).</td>
</tr>
<tr>
<td><strong>Substitution</strong>&lt;br&gt;Recall&lt;br&gt;Remembering Comparing</td>
<td>Technology acts as direct tool substitute with no functional improvement. Student work can be completed without the use of the technology. No significant change in the tasks students are being asked to complete.</td>
<td>Student work can be completed without the use of the technology. No significant change in the tasks students are being asked to complete.</td>
</tr>
</tbody>
</table>


Hamilton County Department of Education, 2014
Section 3: Effective Strategies

Math Talks……………………………………………………………………………………………………36
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  Visual Routines ..................................................................................................................43
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Toolkit Strategy: Math Talks

Grade Span: K-12
Math Practice: MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.8 Look for and express regularity in repeated reasoning.

Purpose: This strategy helps to develop quality student discourse in a whole class setting as students are encouraged to explain their thinking, justify their reasoning, and make sense of each other’s strategies. The teacher steps away from his/her role of authority, and into the role of facilitator by asking students questions, recording student responses on the board, and encouraging students to make meaning out of the mathematics through verbal exchange.

Protocol:
1. Teacher launches a mental math problem (10-15 minutes total time)
2. Give silent think time to students. Let them know they will need to explain their reasoning.
3. Ask students to share answers with a partner
4. Ask for a volunteer to their answer with the whole class
   a. Scribe every response on the board, even if it’s incorrect
   b. Question offerings for teachers:
      i.  Tell me more about why you think ....
      ii. Did anyone else get that answer, but for a different reason?
      iii. Can anyone come up with a justification for ____?
5. Close activity and thank students for their participation
Variations:
- Possible Math Talks:
  - Number Talk
  - Math Talk
  - Which one does not belong
- To start, do not jump into the math. Start with a more “get to know you” type of number talk, like a dot talk, or a which one does not belong prompt about logos or food to introduce the structure and just get students talking
- If students struggle to share their ideas, use sentence stems like the ones listed on page 41 of this document
- If students do not voice many different strategies try asking questions like, “How might someone else see solving this problem?”
- Use page 20 or 21 of the Oakland Unified School District Instructional Toolkit for Mathematics as a great planning resource

Toolkit Strategy: 4 Quadrant Maths
Grade Span: K-12
Math Practice: MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively
MP4: Model with mathematics.

Purpose: The 4-quadrant maths template is referenced from YouCubed. The template expects students to create a mathematical model, apply methods, draw diagrams, connect ideas, reason about connections, and communicate in different forms. Students can use a 4-quadrant template to make sense of a problem and guide them through this process.

Students learn to:
- Make sense of mathematics in their own way
- Reason abstractly and quantitatively
- Make sense of computation
- Use appropriate tools strategically
- Model with mathematics

Protocol:
1. Teacher provides blank paper and asks students to fold it into 4 quadrants and fold the center corner to make a triangle in the center of the paper. Teacher can also provide the template preprinted.
2. The teacher provides a problem to the students, which is written in the center diamond. The students solve the problem using a verbal representation, a picture or a doodle, a real life situation/application, computation/procedure, and an expression/equation.
Toolkit Strategy: 4 Quadrant Maths

Variations:
Teacher can provide any information that belongs to either of the following:

- verbal representation
- a picture or a doodle
- real life situation/application
- computation/procedure
- expression/equation

The students then work backward and complete the quadrants.
Toolkit Strategy: Quality Questioning

Grade Span: TK-12

Math Practice: MP 3: Construct viable arguments and critique the reasoning of others and
MP 6: Attend to precision,

Purpose: Teacher engages students in questioning that help them delve deeper into their thinking. Students learn to question each other and themselves, becoming a form of metacognitive learning.

Students learn to:
- Make sense of mathematics in their own way.
- Make mathematically convincing arguments through questioning.
- Critique and build on ideas of their classmates.
- Persevere and solve complex problems.
- Reflect on their mathematical process and final solution to determine if solution makes sense.

Protocol: When students are working on a task, teacher circulates and asks questions that deepen a student’s thinking. During the different stages of instruction, questions are posed to facilitate and deepen student thinking. Students can also ask the questions with one another when working together and analyzing the work of others.

<table>
<thead>
<tr>
<th>Getting Started</th>
<th>While Working</th>
<th>Reflecting About the Solution</th>
<th>Responding to Help Clarify Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is this problem about?</td>
<td></td>
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<tr>
<td>Would you please explain that in your own words?</td>
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<tr>
<td>What do you (need to) know?</td>
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</tr>
<tr>
<td>How can you get the information?</td>
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<tr>
<td>What do you need to find out?</td>
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<tr>
<td>Where can you begin?</td>
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<tr>
<td>What terms do you understand or not understand?</td>
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<tr>
<td>Have you solved similar problems that would help?</td>
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</tr>
<tr>
<td>What problems have you solved that are similar?</td>
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<tr>
<td>What have you already tried?</td>
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<tr>
<td>How can you organize the information?</td>
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<tr>
<td>Can you make a drawing/model to explain your thinking?</td>
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<tr>
<td>What would happen if…?</td>
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<tr>
<td>Can you make a prediction?</td>
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<tr>
<td>Are there other possibilities?</td>
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<tr>
<td>Do you see any patterns or relationships that might help you solve this?</td>
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<tr>
<td>Can you describe an approach/strategy you can use to solve this?</td>
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<tr>
<td>How do you know your solution/conclusion is reasonable?</td>
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</tr>
<tr>
<td>Have you answered the question?</td>
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<td></td>
</tr>
<tr>
<td>How do you know you have solved the problem?</td>
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<td></td>
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<tr>
<td>How did you arrive at your answer?</td>
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<tr>
<td>How did you know you were finished?</td>
<td></td>
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<tr>
<td>How can you convince me that your answer makes sense?</td>
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<tr>
<td>What did you try that did not work?</td>
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<tr>
<td>Is that the only possible answer?</td>
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<tr>
<td>Can you describe an approach/strategy you can use to solve this?</td>
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</tr>
<tr>
<td>How can you make your explanation be made clearer?</td>
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</tr>
<tr>
<td>Tell me more.</td>
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</tr>
<tr>
<td>Can you explain that in a different way?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is there another possibility or strategy that would work?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Would this method work in other problems with different numbers?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Help me to understand this part…</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is there a more efficient strategy?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is there another strategy that would work?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How could you help another student without telling them the answer?</td>
<td></td>
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</tr>
</tbody>
</table>
The Art of Questioning in Mathematics
The following is a set of questions, which may be helpful in guiding inquiry as an instructional strategy to deepen and expand students’ mathematical thinking and problem-solving abilities.

<table>
<thead>
<tr>
<th>Help student work together to make sense of mathematics</th>
<th>Help student learn to reason mathematically</th>
<th>Help student learn to conjecture, invent, and solve problems</th>
<th>Help student connect mathematics, its ideas, and its applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Who agrees/disagrees? Who will explain why or why not?</td>
<td>• Does that always work? Why or why not?</td>
<td>• What would happen if ___________? What if not?</td>
<td>• How does this relate to ___________?</td>
</tr>
<tr>
<td>• Who has the same answer, but a different way to explain it?</td>
<td>• Is that true for all cases? Explain.</td>
<td>• Do you see a pattern? Explain.</td>
<td>• What ideas that we have learned before were useful in solving this problem?</td>
</tr>
<tr>
<td>• Who has a different answer? What is your answer and how did you get it?</td>
<td>• What is a counterexample for this solution?</td>
<td>• What about the last one?</td>
<td>• What problem have we solved that is similar to this one? How are they the same? How are they different?</td>
</tr>
<tr>
<td>• Please ask the rest of the class that question.”</td>
<td>• How could you prove that?</td>
<td>• How did you think about the problem?</td>
<td>• What uses of mathematics did you find in the newspaper last night?</td>
</tr>
<tr>
<td>• Explain to your partner your understanding of what was just said.</td>
<td>• What assumptions are you making?</td>
<td>• What decision do you think he/she should make?</td>
<td>• What example can you give me for ___________?</td>
</tr>
<tr>
<td>• Convince us that makes sense.</td>
<td></td>
<td>• What is alike and what is different about your method of solution and his/hers? Why?</td>
<td></td>
</tr>
</tbody>
</table>

(Excerpt from NCTM Professional Teaching Standards)
Toolkit Strategy: Quality Questioning

Variations:
Questions are tiered based on student need.
Questions are also based upon where students are in their problem solving process.
Students can ask questions to their peers when working as a group
Students are encouraged to ask these questions to their peers when presentations are given or work is reviewed
Toolkit Strategy: Number Sense Routines: Visual Routines

Grade Span: K-6
Math Practice: MP 7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning
Purpose: Teacher engages students in routines that help build number sense, numerical literacy, and understanding of numbers.
“These number sense routines are not “auto pilot” activities, but opportunities for meaningful practice.

- Provide daily number sense experiences
- Include discussion about numbers and their relationships
- Respond to students’ current understandings
- Build on students’ existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking
- Help students make connections to big ideas in mathematics

-Jessica F. Shumway, author of Number Sense Routines

Visual Routines help student in seeing and Conceptualizing Quantities, develop their understanding of quantity, give meaning to numbers, understand and see relationships of numbers, understand the operation of numbers

Protocol:

Quick Images Using dot Cards: “These are cards with dots on them arranges in various groups. You can make your dot cards based on twos, fives, tens, doubles, or the visual arrangement of dice or dominoes. You flash the amount quickly, giving students 3-5 seconds to visualize the amount. Then, you ask students what they saw. This will encourage them to think in groups rather than count by ones.”

Ten Frames: You can use ten-frames much like quick images. The difference in using the ten-frame is that the five- and ten- structures are highlighted by the configuration of the frame. The ten-frame can better highlight the idea of teen numbers- the concepts that a teen number is a ten and then some more.

Rekenrek: The rekenrek is a Dutch arithmetic rack. It has two rows with 10 beads on each (or, on a rekenrek with 100 beads, then rows with 10 beads on each row). Each row of 10 beads is made up of 5 red and 5 white beads. There is a white panel attached to the end of the frame that allows you to hide some beads and show other beads. You can use the rekenrek in a quick Images manner to encourage the use of grouping. And like the ten frame, the rekenrek highlights the five- and ten- structures. The rekenrek is different in that it has 20 beads total (or 100 total) and the beads move on the rods giving it a kinesthetic aspect.

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F. Shumway
Toolkit Strategy: Number Sense Routines: Visual Routines

Variations:
Dot Cards:
Dots on dot cards can be same color or shading, different shadings, or scattered.
“This more scattered arrangement in combination with the speed of the routine forces students to find their own quick ways to group the amount so they can identify the total.”

Ten Frame Routines:
Combinations of 10,
Amounts in teens,
Compensation strategies,
Part-part-whole with 20 being the whole amount,

Rekenreks:
Use 20-100 bead rekenrek for higher numbers

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: Number Sense Routines: Counting Routines

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Math Practice: MP 7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning

Purpose: Teacher engages students in routines that help build number sense, numerical literacy, and understanding of numbers.

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- Help students make connections to big ideas in mathematics

-Jessica F. Shumway, author of Number Sense Routines

Counting Routines help students understand place value and the number system

Protocol:

Count Around the Circle: “Choose a counting sequence- for example, count by tens starting at thirty-two- and go around the circle as each person says a number. (For example, the first one says, “thirty-two”, the second person says “forty-two”, the next person says, “fifty-two”, and so on.)”

Choral Counting: “In this routine, the class counts aloud a number sequence all together.”

Start and Stop Counting: “The class counts a number sequence all together, with a starting number and a stopping number. For example, have the class count by tens, starting with 26 and stopping at 176. In addition to whole class, this routine, works particularly well with small groups and individual students.”

Organic Number Line: “This is a number line that you can add to continuously throughout the year. Think of it as one section of your “whole number” number line- you are magnifying (and hence adding more details to) the number line from 0 to 2. For example, there are many numbers that fall between 0 and 1: ½, ¼, ¾, .025, 0.3333, etc. There are also different ways to represent each of these numbers, and some of those numbers are equivalent.”

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: Number Sense Routines: Counting Routines

Variations:
Counting around the circle:
Count by ones, tens, fives, twos, threes, etc., starting at zero.
Count by ones, tens, fives, twos, threes, etc., starting at various points.
Count by tens, starting from 320 (320, 330, 340, 350, 360, 370 . . ).
Count by tens, starting from 53 (53, 63, 73, 83, 93, 103, 113. . ).
Count backward by ones, tens, fives, and twos, starting at various points.
Count backward by tens, starting from 110 (110, 100, 90, 80 . . ).
Count backward by tens, starting from 322 (322, 312,302, 292 . . ).
Count by halves (0, ½, 1, 1½, 2, 2½, 3 . . .), starting at zero or at various points (16, 16½, 17,
17½, 18, 18½, 17, 18 . . ).
Count by fourths, eighths, thirds, or sixths, starting at zero or at various points.
Count by wholes, starting at a fractional number.
Count by hundreds, thousands, or millions, starting at zero or at various points

Various Questioning types

Choral Counting
Kinesthetic Choral Counting
Small group counting

Start and Stop Counting:
Counting on and then back, counting with decimals and fractions, counting with odd/even
patterns

Organic Number Line:
Ask different questions based on what the number line is focused on. For example, ask questions
based on benchmarks, equivalency, whole and parts of the whole, doubling, and halving.

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: Number Sense Routines Playing with Quantities

Grade Span: K-6
Math Practice: MP 7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning
Purpose: Teacher engages students in routines that help build number sense, numerical literacy, and understanding of numbers.
“These number sense routines are not “auto pilot” activities, but opportunities for meaningful practice.
- Provide daily number sense experiences
- Include discussion about numbers and their relationships
- Respond to students’ current understandings
- Build on students’ existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking
- Help students make connections to big ideas in mathematics

-Jessica F. Shumway, author of Number Sense Routines

Playing with quantities helps make sense of numbers and relationships. Students are encouraged to play with quantities, breaking them apart and putting them back together, as well as think about how numbers are composed and how the base ten place-value system works. (Shumway 81)

Protocol:
Teacher creates a number using various models (ten wands, drawing, story, and writing). Students think of strategies for composing and decomposing the number. Questions are asked to students to help them think flexibly and see relationships between the various place values.

Ten Wand: A ten wand is made up of ten Unifix cubes, five of one color and five of a different color. The wand breaks in two pieces at various places (decomposing the ten) to help students see combinations visually.
Ways to Make a Number: Students write as many ways at they can think of to “make” a selected number. They might use visuals of the quantity, equations, models, and so on.
Today’s Number: The teacher chooses a number, such as ten to be Today’s Number (there are a variety of reasons for picking a particular number) and asks various questions about the number, such as: “When is ten big? When is ten small?”
Mental Math: Present an equation or story problem and ask students to solve it in their heads (without paper and pen or manipulatives). Children should then verbalize the strategies they used mentally.

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: Number Sense Routines Playing with Quantities

Variations

Ten Wand:
- Combinations of Ten
- Part-Part Whole
- Commutative Property
- Using 5 and 10 Structure

Ways to Make a Number:
- Routine can be open ended
- Routine can have constraints

Today’s Number
- Ask various questions for that day’s number

Mental Math routines for playing with quantities
- Decomposing numbers
- Compensation
- Making Tens
- Counting On
- Counting Back
- Number Lines

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: Number Sense Routines: Calendar and Data Routines

Grade Span: K-6
Math Practice: MP 7: Look for and make use of structure
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Purpose: Teacher engages students in routines that help build number sense, numerical literacy, and understanding of numbers.
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• Encourage students to play with numbers and enrich their mathematical thinking
• Help students make connections to big ideas in mathematics

-Jessica F. Shumway, author of Number Sense Routines

Calendar routines provide authentic times to use number sense. As classes discuss the calendar or data they have collected, the focus of the discussion is not on the mathematics. The mathematics is embedded in the discussion- it is being applied and used authentically within a context. Discourse is at the heart of calendar time. It is a time for children to talk about what they see, clarify their thinking, share their unique strategies, and develop understanding over time. (Shumway 101)

Protocol:

Calendar: “Use a real calendar in addition to a premade calendar from the teacher store. As a class, write in important days throughout the school year (birthdays, field trips, etc). Integrate social studies and science.” Teacher asks various questions about calendar dates and days. Students analyze days, occurrence of dates, and difference of dates of events.

Collecting Data Over a Long Period of Time: “Collect data, such as temperature, weather, and sunrise/sunset times, over time on graph paper in public spaces in the classroom on a daily basis. Once or twice a month, hold class discussion about the data trends and the interpretation and analysis of the data.” Teacher leads discussion around temperature, weather, statistics, and patterns in the data collected.

Counting the Days in School: “Use sentence strips and sticky notes to build a number line throughout the year that will emphasize each tenth day of school.” Teacher helps students discuss the growing quantities and patterns in keeping track of the days in school.

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: Number Sense Routines: Calendar and Data Routines

Variations:

Have an End-of-Month Ceremony with whole class or small group:
Ask a variety of questions:
  Recall prompts & questions
  ● “Find the ____”
  ● “Find the number that represents this amount.” (Show dot card)
  ● “Find the number one less/one more than ____.”
  Open-ended questions
  ● “Find a number greater than/less than/close to ____”
  ● “What patterns do you notice? How does that help us?”

Use Twelve-Month Calendar to extend questions

Collecting Data Over a Long Period of Time:
  ● Use a variety of graph types: line graph, bar graph
  ● Summarize Data with Mode, Average, and Range
  ● Bring in discussion of science concepts to think more deeply about why temperature changes
  ● Assign Data Collection Jobs: Data Assistant, Weather Monitor, Daytime Tracker, Moon watcher, Calendar Keeper, Data Supervisor

Counting the Days in School:
  ● Use different models: linear, array (hundreds chart), manipulatives (counting cubes in sticks of 10)
  ● For older students, use the calendar in conjunction with the days in school (consider weekends and days not in school)

Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica F Shumway
Toolkit Strategy: 3 Reads +

Grade Span: 3 - 12
Math Practice: MP1: Make sense of problems and persevere in solving them.
Purpose: Through the 3 Reads + process, students practice a process and structure for tackling word problems. They practice annotation notes for reading in mathematics while identifying a question within a word problem. Students are encouraged to reflect on their mathematical process and final answer to determine if answer makes sense within the context

Protocol:
Read #1: Skim
- Ask for pencils down
- Read through the entire text quickly and get an idea of what it is about, “the gist.” It’s not important to understand all of the details. DO NOT ANNOTATE THE TEXT THIS TIME.
- Model what it looks like to read for the gist
- Have students paraphrase the story to one another.

Read #2: Actively Annotate
- Read through the entire text again and annotate using the following codes posted for the students to see (teachers, feel free to add your own codes!)
  - Circle words you don’t know and place a ? in the margin
  - Underline important math information
  - Summarize in the margin
- Model how to annotate and think out loud while you give an example of annotating. Ex: “I’m underlining _____ and writing _____ in the margin because I think it will be it will help me find information faster later.”
- Give time as a team or class to go over any unknown vocabulary and important information students found. Try a think-pair-share to help students feel comfortable sharing what they don’t understand

Read #3: Identify the Question
- Read through the entire text and highlight the question(s) of the task you will need to answer
- Identify what is known: quantities, prices, measurements, temperatures, etc. Encourage students to draw a diagram, make a table, or a bulleted list
- Identify what we need to know, but is unknown: quantities, prices, measurements, temperatures, etc. Consider using sentence stems to help students.
- As a team or class go over the question(s) of the task

Read # 3+: Reflect in Context
- After finding the answer, ask students to reflect on:
  - Have they answered the question they identified in Read #3?
  - Does the answer makes sense given the context explored in Read #2?
Toolkit Strategy: 3 Reads +

Variations:
Graphic Organizer
● A vocabulary graphic organizer, like the Frayer Model, during Read #2 may help students learn new vocabulary

Sentence Stems for Classroom Discussion
● After Read #2, give sentence stems during the class discussion to help students respond to each other in respectful and productive ways. For example:
  o I agree with _____ and also thought ____ was an important piece of information.
  o My summary of this problem or context is _____.
  o _____ can you tell me more about why you think _____ is important information?
● After Read #3, give sentence stems to help students discover and discuss the question. For example:
  o This problem is similar to _____.
  o The first thing I’m going to do is _____.
  o If my strategy isn’t working, then I will _____.

Questions that promote student to student discourse
● After the reflective Read #3+ ask students to question each other’s answers. For example:
  o Who has the same answer, but solved it in a different way?
  o Tell me what ____ means.
  o Why is that the answer?
  o I agree/disagree with ____’s answer because _____.

Additional Annotations for Read #2
● Star – Important
● Check Mark – I understand
● Question Mark – I don’t understand
● Infinity Symbol – I made a connection
● Exclamation Mark – I’m surprised
Section 4
Content Literacy

Section 4: Content Literacy

Teaching and Building Academic Language..........................................................54
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Word Wall Model.................................................................................................61
In order to succeed in mathematics, students must be able to demonstrate academic and linguistic abilities for high levels of learning and thinking (Mathematical Practice Standard 3). Mathematical Practice Standard 4 describes how students are expected to interpret meaning from words, symbols, data, and/or visuals (Zwiers 2014, page 103).

In order to support our students with the above two expectations teachers of mathematics must be teachers of the language and vocabulary of mathematics. According to Dr. Jeff Zwiers (Stanford University) in his book *Building Academic Language* teaching the language of math is more challenging than any other subjects for several reasons:

- There is less overlap with concepts, ideas, and terms. Math has distinct vocabulary terms.
- Students must learn to decipher and use a wide range of symbols.
- Students must learn to read the problems many ways, not just left to right.
- Math texts have a denser concentration of abstract concepts than other academic texts.
- Math concepts are embedded within other math concepts: they depend on prior knowledge and experience.
- Historically there has been a lack of extended student talk about math in math classes.

All teachers are teachers of reading, writing, listening, and speaking within each content class. Academic language has to be taught in a systematic structure in order to build competent thinkers from year to year.

Whether a student is English Only or an English Language Learner, he/she must learn the total vocabulary/language (symbols, terms, directions, visuals) of mathematics. According to Dr. Zwiers’ academic language is “the set of words, grammar, and discourse strategies used to describe complex ideas, high-order thinking process, and abstract concepts.” Where does a teacher begin in math when symbols, terms, directions, and visuals all must be understood for our students to be successful in math? The operative words are start the process. (See Principles for the Design of Mathematics Curricula: Promoting Language and Content Development, [http://ell.stanford.edu/sites/default/files/u6232/UPSCALE](http://ell.stanford.edu/sites/default/files/u6232/UPSCALE))

**Beginning Steps for Teaching Academic Language**

1. Decide whose method of teaching academic vocabulary you are going to use by researching the following experts online, buying one of their books, or going to training. Some recommended experts are Dr. Jeff Zwiers, Dr. Kate Kinsella, and Dr. Robert Marzano. There are also experts at the Santa Clara County Office of Education as well as in your district.
2. Understand the difference between Tier 1, Tier 2, and Tier 3 math content words (see Academic Vocabulary Chart).
3. Read each module/chapter in your adopted curriculum for the vocabulary to be taught. Identify the Tier 2 words that you will directly teach. Identify the Tier 3 words that you will explain and illustrate in the context of a lesson.

4. Put up a mathematics word wall in your classroom. Display these words on the word wall after using Dr. Kate Kinsella’s explicit vocabulary teaching routine or Dr. Robert Marzano’s six step routine. (See word wall model described in this article.)

5. Students should have a learning log, math journal, or math vocabulary dictionary to write the expanded vocabulary definitions.

6. Use games for students to play to practice vocabulary.

7. Provide rich mathematical tasks for partner and group work so students must use the vocabulary to explain and discuss the task. Go to http://resources.corwin.com/VL-mathematics for a rubric describing rich mathematic tasks or buy the book Visible Learning for Mathematics by John Hattie, Douglas Fisher, and Nancy Frey for more explicit support.

8. Make sure students have opportunities to use these words in writing.

Additional Resources for Teaching Math Language

Engage New York has an accompanying guide to each grade level module called Scaffolding Instruction for ELLs: Resource Guide for Mathematics. This guide was developed by the American Institutes for Research. The guide describes the following techniques recommend by Moschkovich (2014) and Civil (2007) which can be used for all students in order to teach math language:

- Teach academic vocabulary.
- Integrate oral and written language instruction.
- Use concrete and visual models.
- Use graphic organizers.
- Use word walls/math journals.

Adding to these recommendations are classroom structures/activities that stimulate mathematical speaking, writing, and thinking from Dr. Jo Boaler’s Mathematical Mindsets, https://www.youcubed.org:

- Select rich mathematical tasks to engage students.
- Develop hands-on experiences.
- Use real-life applications in the curriculum.
- Provide opportunities to work together; use cooperative learning strategies.
- Value mistakes.
- Develop student self-awareness and responsibility.
- Teach math as a subject of patterns and connections.

The instructional materials for teaching mathematics are also resources for teaching math language, but some of these materials have limited information when identifying and describing how to teach math vocabulary. Many do not directly list the module, chapter, or unit vocabulary needed to teach the math standard. Most do not list the unique vocabulary necessary to unpack story problems.
Resources for Beginning Steps for Teaching Academic Language-Word Wall and Direct Teaching of Vocabulary

The power of a word wall in a classroom depends upon making the word wall a living display and making sure that the display has a meaningful connection to the current curriculum (https://www.responsiveclassroom.org/our-classroom-walls/). Displays throughout the classroom need to be fresh, useful, easily visible, and uncluttered.

Ideas for creating a word wall (google Jennifer Cronsberry):

- Mount the words on construction paper or card stock and laminate.
- Color code the words/symbols.
- Use a wall area that is visible to all students. Students need to be able to glance at the word wall from their desks while they are working.
- Mount words on the word wall in alphabetical order. Using alphabetical order makes it easier to skim the list and find the words.

Direct Teaching of vocabulary (Kate Kinsella, https://scoe.org/docs/ah/AH_kinsella2.pdf):

- Guide students in reading and pronouncing the word a few times.
- Have students cap/tap out the syllables for the polysyllabic words.
- Direct students to copy the word correctly.
- Explain the meaning using familiar language (including Spanish cognate).
- Provide two examples within student’s experiential realm.
- Enter the word into the student’s math dictionary using one of the Kinsella models which includes syllabication, part of speech, cognate; definition/meaning; examples within a sentence.

Recommendations for English Language Learners

- For schools with many Spanish speaking students, teach the Spanish cognates (words having the same linguistic derivation as another word or root) when teaching specific English math vocabulary.
- Pair students with bilingual buddies to enhance learning while students are acquiring English.
- Use a cooperative learning structure that enhances mathematical conversations and stimulates participation from low status students.
- Give students access to math hard copy or online dictionaries (with illustrations as part of the definition).
- Use ELD and/or SDAIE strategies for targeted group instruction during the math period.
Additional Resources for English Language Learners

- “Identifying Spanish-English Cognates to Scaffold Instruction for Latino ELs” (2011) describes the importance of developing a cognate recognition strategy for Spanish speaking ELLs.
- Information from Improving Education for English Learners: Research Based Approaches (2010) is from the California Department of Education.
- Use www.wordsift.org for reference and support (Stanford University).
- Various materials developed by Kate Kinsella such as her Academic Vocabulary Toolkits. See Toolkit Strategy: Academic Language Frames/Sentence Starters for Mathematical Discourse.
- Various materials/books from Marzano and Pickering including Marzano’s 6 Step Process for Building Academic Vocabulary.
- Review the resources at www.elachieve.org

Ideas for Struggling Math Students

Every teacher is concerned when students come to them working below grade level. There are three acknowledged methods currently used to help struggling students’ bridge the gap: **intervention, re-teaching, and re-engagement**. An intervention is extended time for students to learn math usually after school, on Saturdays, or during the summer. An example of this type of intervention is the summer Elevate program sponsored by the Silicon Valley Education Foundation. Re-teaching usually consists of teaching the unit again, addressing missing basic skills (including the re-teaching of vocabulary), doing the same problems over, and more practice to learn procedures. Re-engagement is revisiting student thinking, addressing conceptual understanding, examining a task from different perspectives, using a critiquing approach in order to make connections (see https://www.sfusdmath.org/reengagement.html).

In addition to the above three methods **differentiation** within the classroom/course is a process by which differences between learners are accommodated so that all students in a class have the best possible chance of learning (bbactive.schools@pearson.com). Differentiation can take many forms: modified materials, target groups, and mixed ability cooperative groups using complex instruction.

**Instructional Terms**

- **Bilingual buddy** is a term for assigning a bilingual student as a mentor to a non-English or limited-English speaking student. The bilingual student needs to be proficient in English and the first language. The teacher must train the bilingual buddy how to mentor as oppose to “speaking for” the student buddy.
- **Differentiation** is a process by which differences between learners are accommodated so that all students in a class have the best possible chance of learning.
- **Graphic organizers for math** are graphic charts and tools used to visually represent and organize knowledge, key concepts, and key ideas.
- **Intervention** is extended time to re-teach or pre-teach students.
Living Word Wall is a collection of words which are displayed in large visible letters on a wall, bulletin board, or other display surface in the classroom. The word wall is designed to be an interactive tool for students. It contains an array of words that can be used during math lessons.

Math dictionary incorporates all the words on the word wall and may be included in a math notebook or learning log.

Math notebook/learning logs help learners practice writing as they collect and organize information, pose and solve problems, and think visually. Learning logs are an efficient way for students to communicate what they do and do not understand.

Re-engagement is a process to review student thinking to address conceptual understanding.

Re-teaching is to teach the same lesson a second time.

Sentence starters/stems are phrases used to scaffold student oral language and writing in order to explain math processes and concepts.

Verbal scaffolding is used to prompt, guide, and support learners by using a variety of questioning techniques that promote higher levels of thinking as students develop their language skills.
# Academic Vocabulary Development Chart

The goal for vocabulary development is for students to be able to use the instructed words in understanding a text/lesson and to recall the words well enough to use them in speech and writing (Mathematical Practice 3, construct viable arguments and critiques the reasoning of others).

<table>
<thead>
<tr>
<th>Tier 1: Basic/High Frequency Words</th>
<th>Tier 2: General High Frequency Academic Words</th>
<th>Tier 3: Specialized Words or Domain Specific Words</th>
<th>Information for Collaboration to Design Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Everyday Words Examples:</td>
<td>Across Content Areas (utility words)</td>
<td>Limited to a Specific Content Area (precise)</td>
<td>- Teach 400 words per year.</td>
</tr>
<tr>
<td>1. clock</td>
<td>Examples:</td>
<td>Examples:</td>
<td>- Teach word families which are groups of related words such as introduce, introduction.</td>
</tr>
<tr>
<td>2. happy</td>
<td>1. product</td>
<td>1. cosine</td>
<td>- Provide frequent multiple encounters over time.</td>
</tr>
<tr>
<td>3. walk</td>
<td>2. reciprocal</td>
<td>2. linear equation</td>
<td>- Teach Spanish cognates.</td>
</tr>
<tr>
<td>4. baby</td>
<td>3. solution</td>
<td>3. circumference</td>
<td>- Teach the functional words and phrases (mortar).</td>
</tr>
<tr>
<td>- These words may require</td>
<td>4. expression</td>
<td>4. trapezoid</td>
<td>- Teach the topic specific words and phrases (bricks).</td>
</tr>
<tr>
<td>direct teaching for an EL.</td>
<td>- Focus instruction on Tier 2 words.</td>
<td>- Agreement across content areas is needed.</td>
<td>- Develop word walls.</td>
</tr>
<tr>
<td>- <strong>8000 word families</strong></td>
<td>- 7000 word families among Tier 2 words.</td>
<td></td>
<td>- Build on what the student already knows.</td>
</tr>
<tr>
<td>among Tier 1 words.</td>
<td>Criteria for selecting:</td>
<td></td>
<td>- Use concept circles to show relationships between words (see Allen book).</td>
</tr>
<tr>
<td></td>
<td>1. Importance and utility</td>
<td></td>
<td>- Use scaffolding to build a temporary structure that assists the learner to build new skills, concepts, or levels of understanding.</td>
</tr>
<tr>
<td></td>
<td>2. Instructional potential</td>
<td></td>
<td>- Provide paired or group structures that allow students to practice using the vocabulary.</td>
</tr>
<tr>
<td></td>
<td>- Agreement across content areas is needed.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Reference: *Bringing Words to Life* (2002) by Beck, McKeown, Kucan


Reference: *Inside Words* by Janet Allen


Reference: *Building Academic Vocabulary* by Marzano/Pickering

Revised August 2017
Toolkit Strategy: Academic Language Frames/Sentence Starters
For Mathematical Discourse

Grade Span: 3-12
Math Practice: MP 3: Demonstrate academic and linguistic abilities for high levels of learning and thinking. MP: Interpret meaning from words, symbols, data, and/or visuals.

Purpose: Mathematical discourse is the way students represent, think, talk, question, agree, and disagree in the classroom. Teachers use language frames to model language for their students in order to create language behaviors their students can use when working cooperatively, discussing their work, and writing.

Protocol:
1. Teach a few language frames at a time. Post the frame(s) in the classroom.
2. Monitor student discussions and writing for their use. Acknowledge student behaviors.
3. Add additional language frames as students expand their content vocabulary.

Acknowledging Others
- My plan is similar to/related to ____’s plan.
- I agree with ____ that…
- My solution builds upon ____’s solution.

Affirming
- That’s a different way. I had not thought about that.
- I agree with ____ because…

Asking for Clarification
- What do you mean?
- Will you explain that again?
- How did you find your answer?

Disagreeing
- I don’t agree with you because…
- I got a different answer than you.
- I see it another way.

Holding the Floor
- As I was saying…
- If I could finish my thought…
- What I was trying to say was…

Offering a Suggestion
- Maybe we could…
- Here’s something we might try.
- What if we…

Reporting a Group Solution
- We decided/agreed that…
- We concluded that…
- Our group sees it differently.
- We had a different approach.

Reporting a Partner’s Solution
- ____ shared with me that…
- ____ pointed out to me that…
- ____ emphasized that…
- ____ concluded that…

Reporting a Solution
- I believe this is the correct answer because…
- My solution shows…
- I can verify my answer by…

Soliciting a Response
- What do you think?
- We haven’t heard from you yet.
- Do you agree?
- What is your answer?

Material from Kate Kinsella, Ed.D.
Factors

Numbers that can be multiplied together to find a product

Factors:
- $4 \times 8 = 32$
- $6 \times 3 = 18$

Spanish: factor
Section 5
Resource Links

Section 5: Resources Links

Math Resource links .................................................................63
# Mathematics Resource Links

<table>
<thead>
<tr>
<th>Name</th>
<th>Grade Level</th>
<th>Link</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Achieve the Core</strong></td>
<td>K - 12</td>
<td>Student Achievement Partners (SAP) is a nonprofit organization dedicated to helping teachers and school leaders implement high-quality, college- and career-ready standards. SAP’s work is grounded in research and evidence, and they are committed to sharing standards-aligned tools and practices freely with educators across the country. This site offers hundreds of math and literacy instructional resources for teachers and school leaders. <a href="https://achievethecore.org/">https://achievethecore.org/</a></td>
</tr>
<tr>
<td><strong>3 Act Math</strong></td>
<td>K - 12</td>
<td>A Three-Act Task is a lesson structure designed specifically to engage children in modeling with mathematics. The activity was created by Dan Meyer, <a href="http://bit.ly/2ewaxOz">http://bit.ly/2ewaxOz</a></td>
</tr>
<tr>
<td><strong>Bowland Maths</strong></td>
<td>6 - 10</td>
<td>Bowland Maths aims to make maths engaging and relevant to pupils aged 11-14, with a focus on developing thinking, reasoning and problem-solving skills. In these materials, the maths emerges naturally as pupils tackle problems set in a rich mixture of real-life and fantasy situations. <a href="http://www.bowlandmaths.org.uk/index.html">http://www.bowlandmaths.org.uk/index.html</a></td>
</tr>
<tr>
<td><strong>Desmos</strong></td>
<td>K - 12</td>
<td>Desmos wants to help every student learn math and love learning math. But “every student” is a lot of students so we create digital math tools and let the Internet take them to anyone who wants them. <a href="https://www.desmos.com/">https://www.desmos.com/</a></td>
</tr>
<tr>
<td><strong>Elementary Mathematics Specialists (ems&amp;tl)</strong></td>
<td>K - 8</td>
<td>The Elementary Mathematics Specialists &amp; Teacher Leaders (ems&amp;tl) Project, sponsored by The Brookhill Institute of Mathematics, addresses issues related to and in support of Elementary Mathematics Specialists (EMS). The Project engages mathematics specialists nationally, work with a cadre of specialists and supervisors to provide that up close and personal view of the everyday challenges in the work of elementary mathematics coaches/specialists. <a href="https://sites.google.com/site/emstlonline/home">https://sites.google.com/site/emstlonline/home</a></td>
</tr>
<tr>
<td><strong>EngageNY</strong></td>
<td>K - 12</td>
<td>The optional curricular materials on EngageNY are designed to be adopted or adapted. Educators will find both PDF and Word versions available for their use. Some lessons provide detailed instructions or recommendations but it is important to note that the lessons are not scripts but rather they should be viewed as guides so that the reader can imagine how classroom instruction could look. <a href="https://www.engageny.org/common-core-curriculum">https://www.engageny.org/common-core-curriculum</a></td>
</tr>
<tr>
<td>Resource</td>
<td>Grade</td>
<td>Description</td>
</tr>
<tr>
<td>----------------------------------</td>
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<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Estimation 180</strong></td>
<td>K-12</td>
<td>Each day of the school year, students have an estimation challenge to do. With each activity, students improve both their number sense and problem solving skills.</td>
</tr>
<tr>
<td><strong>Gapminder</strong></td>
<td>K-12</td>
<td>Get a global perspective through Gapminder! Explore the tools that add sparkles to fresh numbers into your eyes and upgrade your worldview.</td>
</tr>
<tr>
<td><strong>Illustrative Mathematics</strong></td>
<td>K-12</td>
<td>High-quality educational resources for teachers and students to know, use and enjoy mathematics. The website offers comprehensive mathematics curriculum, tasks, lessons plans, professional services, and a format for engaging mathematical discussions.</td>
</tr>
<tr>
<td><strong>Inside Mathematics</strong></td>
<td>K-12</td>
<td>Inside Mathematics provides a resource for educators around the world who struggle to provide the best mathematics instruction they can for their students.</td>
</tr>
<tr>
<td><strong>Julia Robinson Mathematics Festival</strong></td>
<td>K-12</td>
<td>Julia Robinson Mathematics Festivals inspire students to explore the richness and beauty of mathematics through activities that encourage collaborative, creative problem-solving.</td>
</tr>
<tr>
<td><strong>Mathalicious</strong></td>
<td>K-12</td>
<td>Mathalicious lessons provide teachers with an opportunity to teach standards-based math through real-world topics that students care about.</td>
</tr>
<tr>
<td><strong>Mathematics Assessment Project</strong></td>
<td>6-12</td>
<td>Classroom Challenges are lessons that support teachers in formative assessment. There are 100 lessons in total, 20 at each grade from 6 to 8 and 40 for ‘Career and College Readiness’ at High School Grades 9 and above. Some lessons are focused on developing math concepts, others on solving non-routine problems.</td>
</tr>
<tr>
<td><strong>Mathematics Design Collaborative</strong></td>
<td>K-12</td>
<td>An effective teacher is the single most important in-school factor that can help students succeed. The site offers support to define and identify what makes a teacher effective, and can help more teachers improve how they support students.</td>
</tr>
<tr>
<td>Math Playground</td>
<td>K - 8</td>
<td>Math Playground has many free resources for First to Eighth graders that includes puzzles, logic games, math games, math videos, lessons, and more. [<a href="http://www.mathplayground.com/">http://www.mathplayground.com/</a>]</td>
</tr>
<tr>
<td>----------------</td>
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<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>My Math Forum</td>
<td>K - 12</td>
<td>Look through over 15,000 questions and answers to find the information you want. There is an endless number of topics for you to go through. [<a href="http://mymathforum.com/">http://mymathforum.com/</a>]</td>
</tr>
<tr>
<td>National Assessment of Educational Progress</td>
<td>4 - 12</td>
<td>The results of the NAEP mathematics assessment present a broad view of what our nation's students know and can do in mathematics. The assessment measures students' knowledge and skills in mathematics and students' ability to apply their knowledge in problem-solving situations. Every two years, students at grades 4 and 8 are assessed in mathematics and reading, and students at grade 12 are assessed periodically. [<a href="https://nces.ed.gov/nationsreportcard/mathematics/">https://nces.ed.gov/nationsreportcard/mathematics/</a>]</td>
</tr>
<tr>
<td>National Council of Teachers of Mathematics</td>
<td>K - 12</td>
<td>The National Council of Teachers of Mathematics supports and advocates for the highest-quality mathematics teaching and learning for each and every student. [<a href="http://www.nctm.org/">http://www.nctm.org/</a>]</td>
</tr>
<tr>
<td>National Library of Virtual Manipulatives</td>
<td>K-12</td>
<td>The National Library of Virtual Manipulatives (NLVM) is an NSF supported project that began in 1999 to develop a library of uniquely interactive, web-based virtual manipulatives or concept tutorials, mostly in the form of Java applets, for mathematics instruction (K-12 emphasis). The project includes dissemination and extensive internal and external evaluation. [<a href="http://nlvm.usu.edu/en/nav/vlibrary.html">http://nlvm.usu.edu/en/nav/vlibrary.html</a>]</td>
</tr>
<tr>
<td>Open Middle</td>
<td>K - 12</td>
<td>This website offers challenging math problems worth solving. [<a href="http://www.openmiddle.com/">http://www.openmiddle.com/</a>]</td>
</tr>
<tr>
<td>Performance Assessment Resource Bank</td>
<td>K - 12</td>
<td>The Performance Assessment Resource Bank is an online collection of high-quality performance tasks and resources that support the use of performance assessment for meaningful learning. Resources include performance tasks, professional development tools, and examples of how schools, districts, and states have integrated performance assessment into their systems of assessment. These resources have been collected from educators and organizations across the United States and reviewed by experts in the field. [<a href="https://www.performanceassessmentresourcebank.org/">https://www.performanceassessmentresourcebank.org/</a>]</td>
</tr>
<tr>
<td><strong>Problems of the Month</strong></td>
<td>K-12</td>
<td>The Problems of the Month are nonroutine math problems designed to be used schoolwide to promote a problem-solving theme at your school. Each problem is divided into five levels of difficulty, Level A (primary) through Level E (high school), to allow access and scaffolding for students into different aspects of the problem and to stretch students to go deeper into mathematical complexity. <a href="http://www.insidemathematics.org/problems-of-the-month">http://www.insidemathematics.org/problems-of-the-month</a></td>
</tr>
<tr>
<td><strong>Thinking Collaborative</strong></td>
<td>K - 12</td>
<td>The mission of Thinking Collaborative is to provide individuals and organizations with the strategies, skills and concepts to establish and sustain structures for thinking and collaborating that result in increased performance and resourcefulness. <a href="http://www.thinkingcollaborative.com/strategies/">http://www.thinkingcollaborative.com/strategies/</a></td>
</tr>
<tr>
<td><strong>Trends in International Mathematics and Science Study (TIMSS)</strong></td>
<td>4 - 12</td>
<td>The Trends in International Mathematics and Science Study (TIMSS) provides reliable and timely data on the mathematics and science achievement of U.S. students compared to that of students in other countries. TIMSS data have been collected from students at grades 4 and 8 since 1995 every 4 years, generally. <a href="https://nces.ed.gov/timss/">https://nces.ed.gov/timss/</a></td>
</tr>
<tr>
<td><strong>Tuvalabs</strong></td>
<td>K - 12</td>
<td>Access Tuvalabs to support data and statistical literacy for teachers and students. Tuva promotes three dimensional science learning, and builds conceptual understanding of essential mathematics and statistics concepts. <a href="https://tuvalabs.com/k12/">https://tuvalabs.com/k12/</a></td>
</tr>
<tr>
<td><strong>Understanding Proficiency</strong></td>
<td>3 - 12</td>
<td>Understanding Proficiency provides resources that guide educators in analyzing student work on performance tasks in order to develop a deeper understanding of the Common Core State Standards in mathematics. <a href="https://understandingproficiency.wested.org/">https://understandingproficiency.wested.org/</a></td>
</tr>
</tbody>
</table>
| **Strategic Education Research Partnership (SERP)** | K - 12 | Algebra by Example [http://math.serpmedia.org/algebra_by_example/](http://math.serpmedia.org/algebra_by_example/)  
The 5x8 Card [http://math.serpmedia.org/5x8card/](http://math.serpmedia.org/5x8card/) |
<p>| <strong>When Math Happens</strong> | K - 12 | The site contains numerous resources on growth mindset, 3-Act Math, sample curricula for Algebra and Geometry, blogs and articles on mathematics, and others. <a href="https://whenmathhappens.com/3-act-math/">https://whenmathhappens.com/3-act-math/</a> |</p>
<table>
<thead>
<tr>
<th>Website</th>
<th>Grade</th>
<th>Description</th>
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<tbody>
<tr>
<td>Which One Doesn't Belong?</td>
<td>K-12</td>
<td>This is Which One Doesn't Belong?, a website dedicated to providing thought-provoking puzzles for math teachers and students alike. There are no answers provided as there are many different, correct ways of choosing which one doesn't belong. Enjoy! <a href="http://wodb.ca/">http://wodb.ca/</a></td>
</tr>
<tr>
<td>Wolfram Alpha</td>
<td>K-12</td>
<td>Using this computational knowledge engine, you will be able to solve a lot of math problems in just a few seconds. Mainly, it is a smart calculator that will solve the problem but won't give you any broad explanations for free. <a href="http://www.wolframalpha.com/">http://www.wolframalpha.com/</a></td>
</tr>
<tr>
<td>YouCubed</td>
<td>K-12</td>
<td>Our main goal is to inspire, educate and empower teachers of mathematics, transforming the latest research on math into accessible and practical forms. We know from research how to teach math well and how to bring about high levels of student engagement and achievement but research has not previously been made accessible to teachers. <a href="http://www.YouCubed.org">www.YouCubed.org</a></td>
</tr>
<tr>
<td>101 Questions</td>
<td>K-12</td>
<td>Can you perplex us? Can you show us something that’ll make us wonder a question so intensely we’ll do anything to figure out the answer, including listen to your lecture or watch your slides? Here’s one way to find out. Upload a photo or a video. Find out how many of us get bored and skip it. Find out how many of us get perplexed and ask a question. <a href="http://www.101qs.com/">http://www.101qs.com/</a></td>
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Section 6: APPENDIX

Effective Mathematics Teaching Practices - NCTM ......................................................... 69
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### Effective Mathematics Teaching Practices

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<th>Description</th>
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<tr>
<td><strong>Establish mathematics goals to focus learning.</strong></td>
<td>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td>
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<tr>
<td><strong>Implement tasks that promote reasoning and problem solving.</strong></td>
<td>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</td>
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<tr>
<td><strong>Use and connect mathematical representations.</strong></td>
<td>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</td>
</tr>
<tr>
<td><strong>Facilitate meaningful mathematical discourse.</strong></td>
<td>Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</td>
</tr>
<tr>
<td><strong>Pose purposeful questions.</strong></td>
<td>Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.</td>
</tr>
<tr>
<td><strong>Build procedural fluency from conceptual understanding.</strong></td>
<td>Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td>
</tr>
<tr>
<td><strong>Support productive struggle in learning mathematics.</strong></td>
<td>Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td>
</tr>
<tr>
<td><strong>Elicit and use evidence of student thinking.</strong></td>
<td>Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td>
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</tbody>
</table>


Teachers’ work is remarkably complex—in particular when the goal is not just to present facts and formulas for students to memorize, but to Teach for the Robust Understanding of Mathematics. Because of this, there is always room for learning and growth, for every teacher regardless of prior training, years of experience, or current successes. Indeed, ongoing learning is the essence of teaching.

Our experience as teachers, coaches, and researchers has been that our most meaningful learning occurs when we interact with others, developing and sustaining relationships that simultaneously challenge and support us. These relationships push us to expand our vision of teaching and learning. They offer perspectives on our work that differ from our own. And they respect our intelligence, skill, and intentions—as well as our need to continually grow. These supportive relationships not only help us to alter our practice but also to deepen our understanding of the complex work we are undertaking.

Unfortunately, much of the professional development that we have experienced has focused less on these aspects of learning and more on “experts” sharing “best practices” that we are supposed to simply import to our own classrooms. Here, we have tried to create a professional development tool that builds on what teachers, coaches, and professional learning communities know.

This is a working document. We hope that reflecting on teaching in the ways suggested here will be productive. We also welcome comments and suggestions for improvement. Please contact Evra Baldinger (evra@berkeley.edu) and Nicole Louie (nllouie@utep.edu) with your feedback.

This Conversation Guide is part of a collection of tools and papers supporting Teaching for Robust Understanding, which are available at http://map.mathshell.org/trumath.php. These tools include a domain-general version of the Conversation Guide as well as an algebra-specific version. More details on each dimension in the framework and the research base behind it are provided on the website. An updated version of this Conversation Guide may also be available online.

Work on TRU is the product of The Algebra Teaching Study (NSF Grant DRL-0909815 to PI Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL-0909851 to PI Robert Floden, Michigan State University), and of The Mathematics Assessment Project (Bill and Melinda Gates Foundation Grant OPP53342 to PIs Alan Schoenfeld, U.C. Berkeley, and Hugh Burkhardt and Malcolm Swan, The University of Nottingham).


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This Conversation Guide represents our best efforts to use research to support teacher learning and growth in a way that accounts for both how people learn and the complexity of teaching practice. Instead of prescribing instructional techniques or tricks, we offer a set of questions organized around five dimensions of teaching identified by research as critical for students’ mathematics learning.

The dimensions are summarized in the table below. Together, they offer a way to organize some of the complexity of teaching so that we can focus our learning together in deliberate and useful ways. They include attention to content, practices, and students’ developing identities as thinkers and learners. There is necessarily some overlap between dimensions; rather than capturing completely distinct categories, each dimension is like a visual filter, highlighting different aspects of the same phenomena in everyday classroom life. We encourage you to think about interactions between dimensions when it is useful for you. The questions on subsequent pages of the Guide will also direct your attention to particular kinds of overlap.

<table>
<thead>
<tr>
<th>The Five Dimensions of Mathematically Powerful Classrooms</th>
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<tbody>
<tr>
<td><strong>The Mathematics</strong></td>
</tr>
<tr>
<td>How do mathematical ideas from this unit/course develop</td>
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<tr>
<td>in this lesson/lesson sequence? How can we create</td>
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<tr>
<td>more meaningful connections?</td>
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<td><strong>Cognitive Demand</strong></td>
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<tr>
<td>What opportunities do students have to make their own</td>
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<td>sense of mathematical ideas? To work through authentic</td>
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<tr>
<td>challenges? How can we create more opportunities?</td>
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<tr>
<td><strong>Equitable Access to Content</strong></td>
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<tr>
<td>Who does and does not participate in the mathematical</td>
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<tr>
<td>work of the class, and how? How can we create more</td>
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<tr>
<td>opportunities for each student to participate</td>
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<tr>
<td>meaningfully?</td>
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<tr>
<td><strong>Agency, Ownership, and Identity</strong></td>
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<tr>
<td>What opportunities do students have to see themselves</td>
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<tr>
<td>and each other as powerful mathematical thinkers? How</td>
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<tr>
<td>can we create more of these opportunities?</td>
</tr>
<tr>
<td><strong>Formative Assessment</strong></td>
</tr>
<tr>
<td>What do we know about each student’s current</td>
</tr>
<tr>
<td>mathematical thinking? How can we build on it?</td>
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</table>
WHAT THE CONVERSATION GUIDE IS FOR

The purpose of this Conversation Guide is not to tell anyone how to teach, but to facilitate coherent and ongoing discussions in which teachers, administrators, coaches, and others learn together. We hope that the questions in the Conversation Guide will support educators with different experiences, different expertise, and different strengths to work together to develop a common vision, common priorities, and common language, in order to collaboratively improve instruction and better support students to develop robust understandings.

The Conversation Guide can be used to support many different kinds of conversations, including (but not limited to):

- Conversations to develop common vision and priorities across groups of teachers (within the same school and/or across different schools)
- Conversations between teachers and administrators and instructional coaches around classroom observations (see also the TRU Observation Guide, available at http://map.mathshell.org/trumath.php)
- Conversations between teachers around peer observations
- Conversations around video recordings of mathematics teaching and learning
- Conversations about planning a particular unit or lesson
- Conversations about a particular instructional strategy or set of strategies
- Ongoing individual reflection

We have found that the Conversation Guide can be useful for facilitating a one-time conversation. But its real power lies in its support for creating coherence across conversations. The Guide can help individuals as well as groups of educators to set an agenda and work on it consistently over time. For example, a teacher team (such as a math department) might decide to spend a semester focusing on issues of Equitable Access to Content (Dimension 3). Meeting time might then be spent reflecting on the kinds of access that are currently available to students and planning lessons with the goal of monitoring and expanding access in mind, using the Equitable Access to Content questions and prompts in this Guide. Members of the team might observe each other’s classrooms with a focus on these same questions and prompts. The principal might find ways to support teachers to attend workshops related to the theme of Equitable Access to Content, rather than supporting a series of disconnected trainings.

In the remainder of this document we provide an overview of each dimension; discussion questions for each dimension, for your use in reflecting on and planning instruction; and a set of suggestions for how to use the discussion questions.

We hope you will find the Conversation Guide useful. Happy teaching and learning!
HOW TO USE THIS CONVERSATION GUIDE

Our field tests and our experiences as instructional coaches have led to a few suggestions that may help you make the most of this conversation guide. In this section, we share these suggestions and give some examples of how conversations using the guide might look.

1. **Set a long-term learning agenda.**

Complex learning—like learning how to teach for robust student understanding—has so many facets that it is easy to jump from one thing to another, without making clear progress on anything. Setting a long-term learning agenda can help us focus our energies, whether we’re full-time classroom teachers or people who support classroom teachers. Opportunities to have deep conversations about practice are few and far between, but if we have a core learning agenda that we can return to again and again, we stand a better chance of leveraging all our strengths to learn together about something that matters.

The process of setting an agenda can unfold in many different ways. Various stakeholders may come in with clear (and perhaps competing) ideas about what they want to focus on, or it may happen that no one has a particular preference. Whatever the case may be, it is important that all participants, especially classroom teachers, feel connected to the learning agenda. Our learning is much more powerful when we get to learn about things that trouble or inspire us.

Some examples of long-term learning agendas might be, “This semester, I want to focus on getting students to share their reasoning, not just answers or steps,” or “This year, I want to get better at engaging students who get frustrated and give up easily.” As you set your own learning agenda, it may be useful to read through the dimensions and discussion questions, to see if anything jumps out as particularly important or exciting.

2. **Use the discussion questions like a menu. Pick and choose.**

You might have noticed that there are a lot of questions in this Guide! Our design assumes that you WILL NOT try to discuss every bullet, one by one, each time you use the guide. Instead, we hope you will identify areas of the guide that are appropriate for your learning agenda and return to these areas again and again. We expect that some of the questions will be difficult to answer—and that by discussing them together you will find new ways of understanding teaching and learning and come up with ideas for things to try in order to improve both.

3. **Ground discussion in specific, detailed evidence.**

We’ve all made statements like, “My kids seem to really get linear equations” or “They’re really struggling with fractions.” While these statements convey a picture of student understanding in a quick and concise way, they need to be followed up with more detailed information. Otherwise, it is difficult to make instruction responsive to student thinking, and easy to miss opportunities to build on students’ strengths or address their misconceptions. One way to make our observations more specific is to talk about content with as much detail as possible; for example, instead of saying “My kids are really struggling with fractions,” you might observe that “Even though I’ve seen my kids do just fine with finding equivalent fractions and even adding them, they just seem to shut down every time they see a
fraction,” or “they’re reducing fractions in a mechanical way, but they don’t seem to see that 4/6 of a chocolate bar and 2/3 of a chocolate bar represent the same amount.”

Pressing for specific examples also makes observations more accurate and concrete, helping us get away from our general impressions and closer to actual student thinking. Talking about specific students—and ways that their thinking is or isn’t typical of the class—is another strategy. Not only does this strategy give us a more detailed and accurate picture of the thinking that is going on in our classrooms, but it also opens up instructional possibilities. For example, noticing that today, Jessica drew a really helpful picture to represent fractions could lead you to invite Jessica to share her method with the rest of class, creating a learning opportunity that is invisible in “They’re really struggling with fractions.”

Finally, attending to particular students can help us think about patterns of marginalization in society at large (e.g., fewer resources for ELLs, or stereotypes that link race, gender, and mathematics ability), and how our classrooms might work to replicate or counter those patterns for our own students.

If you are able to ground your conversations in shared experiences of the same classroom (from peer observations, co-teaching, instructional coaching, etc.), you will benefit from more eyes and more perspectives on the details of classroom activity. But even if this isn’t possible in any particular conversation, working with evidence of specific students’ thinking and understanding will make your conversation a richer resource for your own learning.

4. If you are conducting a classroom observation, pre-brief.

Classroom observations are generally accompanied by a debrief conversation. Pre-brief conversations can be just as important. If you can, have a conversation prior to each observation. In this conversation, clarify the goals not just for the lesson, but also for the observation of the lesson. Talk about goals for students, so that observers can understand what the teacher is trying to accomplish. Also remind each other of the teacher’s learning agenda so that you can discuss how the observer can be most helpful. We have found this question especially useful: “What do we want to be able to talk about in our debrief conversation?” From there, you might discuss what the observer should be looking for (e.g., recording the questions the teacher asks, or focusing on a particular student), and what kinds of interactions (if any) the observer should have with students.

The pre-brief conversation is one way of capitalizing on the focus and organization that a learning agenda offers. Without it, it’s easy to get distracted during the observation. It’s also easy for the observer to notice things that are not interesting or important to the teacher, which are less likely to help the teacher learn and grow.

The Conversation Guide includes prompts for planning, which can be used for planning observations as well as for planning lessons. Discussing these prompts should bring to the surface ideas about what likely to happen in the lesson, given the tasks students will be given, the participation structures that will be used, and so on. This kind of anticipatory thinking might lead to tweaks in the lesson plan, but just as important, it can establish common focus between the teacher and observer. This adds richness to the debrief after the lesson; everyone can then reflect on the ways that things worked out the way they were intended to, ways they were surprising, and next steps in light of that information.
5. Link planning to reflection and vice versa.

This Guide includes prompts for “planning” and prompts for “reflecting.” We do not mean to suggest that you restrict each conversation to a focus on one or the other; rather, it will be useful to connect these perspectives in many conversations. Reflection is most practical when it leads to next steps, and next steps (planning) should be firmly grounded in reflection on what has already happened. It is worthwhile to make space for thinking about what has already happened without jumping to next steps too quickly, however. Reflecting on the details of what we have observed opens up possibilities for future action that might otherwise remain hidden (as described above). In addition, different people see different things, and sharing our observations can enrich everyone’s understanding of what students have been doing, thinking, and learning.


Our culture often prompts us to focus on our weaknesses, and on the areas where we need improvement. But our strengths are huge assets when it comes to learning and improving our practice. Knowing our strengths supports us to engage with challenges, giving us a starting point to work from and a reason to believe that we can be successful. Identifying teachers’ strengths, making them explicit, and using them as authentic resources for growth can therefore support teachers to think deeply and critically about their practice, to strive for improvement, to actually improve by building on their strengths, and to develop productive relationships with supportive others, all at once.

In practice, this might mean prompting teachers (not just supervisors) to share their observations, interpretations, and ideas for moving forward; creating diverse opportunities to identify what teachers already do well, including planning together, reflecting together, and observing various kinds of interactions with students (e.g., leading discussions, intervening at small groups, and building rapport with individual students); and building next steps around strengths instead of deficits (e.g., working on supporting students who have been reluctant to participate by building on a teacher’s skill at noticing something that each student is good at).
Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This requires identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It requires engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding. And it requires engaging students in authentic performances of important disciplinary practices (e.g., reasoning abstractly and quantitatively, constructing mathematical arguments and critiquing the reasoning of others).

The Mathematics

Core Questions: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections?

Planning

How will important mathematical ideas and practices develop in this lesson and unit? How can we connect the ideas and practices that have surfaced in recent lessons to this lesson and future lessons?

Reflecting

How have we seen students engage with important mathematical ideas and practices? How has this engagement looked and sounded in specific cases?

Things to think about

- What are the mathematical goals for the lesson?
- What connections exist (or could exist) between important ideas in this lesson and important ideas in past and future lessons?
- How do important mathematical practices develop in this lesson/unit?
- How are facts and procedures in the lesson justified?
- How are facts and procedures in the lesson connected with important ideas and practices?
- How do we see/hear students engage with important ideas and practices during class?
- Which students get to engage deeply with important ideas and practices?
- How can we create opportunities for more students to engage more deeply with important ideas and practices?
We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This kind of learning requires that students engage in productive struggle, grappling with difficult concepts and challenging problems. As teachers, we must support students in ways that maintain their opportunities to do this grappling for themselves. Our goal is to help students understand the challenges they confront and persist in solving them, while leaving them room to make their own sense of those challenges.

Core Questions: What opportunities do students have to make their own sense of mathematical ideas? How can we create more opportunities?

What opportunities exist for students to struggle with important mathematical ideas? How are students’ struggles supporting their engagement with important mathematical ideas? How does (or how could) the teacher respond to students’ struggles, and how do (or how could) these responses maintain students’ opportunities to develop their own ideas and understandings? What resources (other students, the teacher, notes, texts, technology, manipulatives, various representations, etc.) are available for students to use when they encounter struggles? Are there more resources we can make available? What resources are students actually using, and how might they be supported to make better use of resources? Which students get to engage deeply with important mathematical ideas? How can we create opportunities for more students to engage more deeply with important mathematical ideas? What community norms seem to be evolving around the value of struggle and mistakes?
All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn’t make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

**Core Questions:** Who does and does not participate in the mathematical work of the class, and how? How can we create more opportunities for each student to participate meaningfully?

**Planning**
- What opportunities exist for each student to participate in the intellectual work of the class? How can we create more opportunities for more students?

**Reflecting**
- Who have we seen participate in the intellectual work of the class? How has this participation looked and sounded in specific cases?

**Things to think about**
- What is the range of ways that students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting text, using manipulatives, connecting different ideas, etc.)?
- Which students participate in which ways?
- Which students are most active, and when?
- In what ways can particular students’ strengths or preferences be used to engage them in the mathematical activity of the class?
- What opportunities do various students have to make meaningful mathematical contributions?
- What are the language demands of participating in the mathematical work of this class (e.g., academic vocabulary, mathematical discourse practices)?
- How can we support the development of students’ academic language?
- How are norms (or interactions, lesson structures, task structure, particular resources, etc.) facilitating or inhibiting participation for particular students?
- What teacher moves might expand students’ access to meaningful participation (such as modeling ways to participate, holding students accountable, point out students’ successful participation)?
- How can we support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.)?
- How can we create opportunities for more students to participate more actively?
Many students have negative beliefs about themselves and mathematics, for example, that they are “bad at math,” or that math is just a bunch of facts and formulas that they’re supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and ownership over their own learning. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

**Agency, Ownership, and Identity**

Core Questions: What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?

Plannin
What opportunities might exist for students to generate and explain their own ideas? To respond to each other’s ideas? How can we create more opportunities?

Reflectin
How have we seen students explain their own and respond to each other’s ideas? What has that looked and sounded like in specific cases?

Things to think about

- Who generates the ideas that get discussed?
- What kinds of ideas do students have opportunities to generate and share (strategies, connections, partial understandings, prior knowledge, representations)?
- Who evaluates and/or responds to others’ ideas?
- How deeply do students get to explain their ideas?
- How does (or how could) the teacher respond to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.)?
- How are norms about students’ and teachers’ roles in generating ideas developing?
- How are norms about what counts as mathematical activity (justifying, experimenting, connecting, practicing, memorizing, etc.) developing?
- Which students get to explain their own ideas? To respond to others’ ideas in meaningful ways?
- Which students seem to see themselves as powerful mathematical thinkers right now?
- How might we create more opportunities for more students to see themselves and each other as powerful mathematical thinkers?
We want instruction to be responsive to students’ actual thinking, not just our hopes or assumptions about what they do and don’t understand. It isn’t always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students’ understandings, partial though they may be, and build on them.

**Formative Assessment**

*Core Questions: What do we know about each student’s current mathematical thinking? How can we build on it?*

Planning

- What do we know about each student’s current thinking, and how might this lesson or unit build on that thinking?
- How can we learn more about each student’s thinking?

Reflecting

- What have we learned in recent lessons about each student’s thinking? How did this thinking look and sound in specific cases?
- How was this thinking built upon?

**Things to think about**

- What opportunities exist (or could exist) for students to develop their own strategies, approaches and understandings of mathematics?
- What opportunities exist (or could exist) for students to share their ideas and reasoning and to connect their ideas to others’?
- What different ways do students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with materials/artifacts, etc.)?
- Who do students get to share their ideas with (a partner, a small group, the whole class, the teacher)?
- What opportunities exist to build on students’ mathematical thinking, and how are teachers and/or other students taking up these opportunities?
- How do students seem to be making sense of the mathematics in the lesson, and what responses might build on that thinking?
- How can activities be structured so that students have more opportunity to build on each other’s ideas?
- What might we try (what tasks, lesson structures, questioning prompts, etc.) to surface student thinking, especially the thinking of students whose ideas we don’t know much about yet?
A NOTE: What We Mean By “Important Mathematical Ideas and Practices”

“Important mathematical ideas” are notoriously hard to define. Which ideas are important? Which are not? What even counts as an “idea”? Who should have the authority to decide? Our intention with the Conversation Guide is to support discussions about these questions rather than to offer answers. To us, it is much more important to work together to push our students and ourselves as educators toward more interconnected and fundamental understandings of mathematics than to decide exactly which ideas are most important. This pushing is crucial, because traditional views of school mathematics—and many of today’s textbooks and standards documents—define mathematics in terms of isolated topics, skills, and sub-skills. Thinking about the progression of mathematical ideas as “Day 1: Add and Subtract Fractions With Like Denominators; Day 2: Multiply Fractions; Day 3: Divide Fractions; Day 4: Add and Subtract Fractions with Unlike Denominators” (a typical textbook progression) makes it difficult to develop conceptual understanding and a sense of meaning behind all of the mechanics. This is both untrue to mathematics as a discipline and alienating for many students.

One way of finding connections among apparently isolated topics is to focus on core mathematical practices. For example, constructing an argument is one such practice. Creating opportunities for students to develop skill in constructing mathematical arguments can bridge the otherwise disparate topics that math courses are typically supposed to cover. (Note the differences between a skill like constructing an argument and a skill like adding fractions.) Yet a focus on core practices does not eliminate the need to identify important mathematical ideas and use these ideas to organize instruction.

We find the questions below useful for shifting our focus from facts and procedures to important mathematical ideas. We hope they will be helpful for you as well.

- What do we want students to understand about the relevant mathematical objects (fractions, negative numbers, the coordinate plane, triangles, etc.) in this lesson? In this unit?
- What mathematical relationships, patterns, or principles do we want students to understand in this lesson? In this unit?
- How might students connect math ideas in this lesson/unit with ideas that came before or will come later? Are there overarching principles or relationships or patterns that they might work toward understanding?
- What are different ways of representing the math in this lesson/unit? How might different representations be connected to each other and how might these connections deepen our students’ understanding?
- How do the ideas we’re considering develop across multiple lessons/units?
- What are some ways to make connections to this idea in different lessons/units/content areas?

Some examples of math ideas that might be considered “important”:

- Area and perimeter are fundamentally different measurable attributes of two-dimensional
shapes. It is possible to change shapes such that neither, one, or both of these attributes change. For some families of shapes, there are interesting relationships between them.

- Relationships between two variables can be represented using equations, tables, graphs, and verbal descriptions. Parameters of the relationship between the variables (e.g., the rate of change) can be identified in each of these representations and connected across representations.

  - Right triangles have special properties that are different from the properties of other triangles. These properties give us special access to information about things like angle measures and side lengths in particular right triangles.

  - Many sets of changing quantities are proportionally related. This means that certain aspects of the relationship are constant and unchanging, which allows us to use the relationship to determine one quantity given the other.

One characteristic of all of these ideas is that they go beyond naming topics and skills. For example, we might know that we want to “cover proportional relationships” in a particular unit, or that we want students to be able to solve proportions. However, without consideration of the important underlying ideas that we want our students to make sense of, we are likely to get lost in facts and procedures. We are likely to miss opportunities to support students to build conceptual understandings, to make connections, and to develop a sense of themselves as powerful learners and thinkers.

Our hope is that as teachers and others think together about teaching, they can continuously push each other to think about the mathematics that students need to learn in bigger, deeper, richer, and more interconnected ways. So while our discussion questions frequently refer to “important mathematical ideas” as though there were a set list of such ideas somewhere that you could simply consult, we hope that you will instead find ways to explore and interrogate what “important mathematical ideas” means to you.
Toolkit Strategy Nomination Template 2017-18

Format: Use 12 font size and Times New Roman font face. Use bullets and numbers only.

<table>
<thead>
<tr>
<th>Your name(s):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy Name:</td>
</tr>
<tr>
<td>Grade span:</td>
</tr>
<tr>
<td>Standard for Mathematical Practice(s):</td>
</tr>
</tbody>
</table>

**Strategy Description:**
Generally describe the strategy and why it is important and relevant to powerful math instruction.

**Basic Structure:**
Describe how to carry out/perform or use this strategy.

**Key Strategy Features:**
There are many ways to implement a strategy. In order to implement the strategy effectively, describe the steps/procedures clearly in order to achieve the desired strategy results for student growth in mathematics. What are the “must dos” and why?

**Feature 1:**

**Feature 2:**

**Feature 3:**

**Feature…**

**Variations:**
Describe some ideas for differentiation/varying the strategy in order to meet the learning needs of English Learners, special needs, and above proficient students. Also add ideas for use with small groups and the whole class.

**Citations/credit:**
Be sure to cite credit if your strategy, full or part, was taken from workshop material, another author/publisher, and/or colleague.

**Process:**
1. Email to ________________________________
2. Toolkit Committee review
3. Possible editing
4. Inclusion in Toolkit